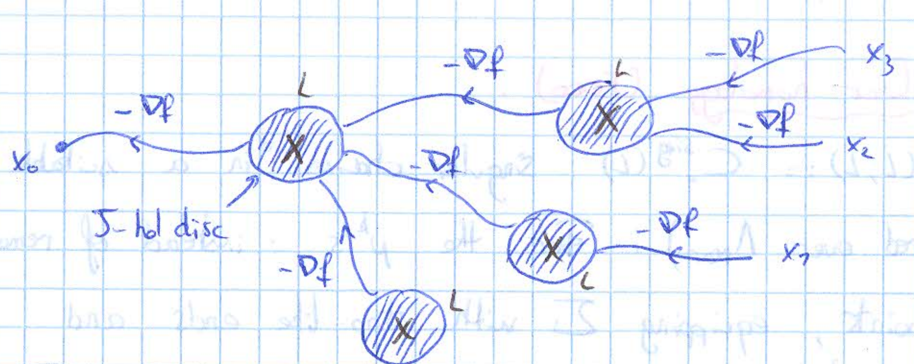


this contributes $T^{\text{area}} x_0$ to $\mu^3(x_3, x_2, x_1)$.

Bubbling of discs is no longer a boundary in these moduli spaces
$\rightsquigarrow$ broken trajectories are boundaries.

09/05/16

<u>One example</u> (very sketchy) of a calculation of monotone $HF^*(L, L)$ with the A_∞-structure: $T^n_{Cl} \subset \mathbb{P}^n$ the Clifford torus.
Define $T^{n+1} = \left(S^1\left(\frac{1}{\sqrt{n+1}}\right)\right)^{n+1} \subseteq S^{2n+1}(1) \subseteq \mathbb{C}^{n+1}$. Then,
$T^n_{Cl} := T^{n+1}/S^1_{\text{diagonal}} \subseteq S^{2n+1}/S^1 \cong \mathbb{CP}^n$.

[Oh]: $L := T^n_{Cl}$ is monotone, wrt ω_{FS}, and $N_L = 2$. This follows from the analysis of ω and μ on $\pi_2(X, L)$, via

$$0 \to \pi_2(X) \to \pi_2(X, L) \to \pi_1(L) \to 0.$$

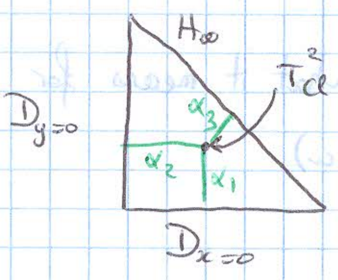
ex: presenting $\mathbb{CP}^2 = \mathbb{C}^2 \cup H_\infty$, T^2 is a standard product torus in each $\mathbb{C}$-coordinate, of radius $\frac{1}{\sqrt{3}}$.

There is a canonical choice of Spin structure on T^n_{Cl}, coming from the canonical trivialization $TT^n \cong T^n \times \mathbb{R}^n$.
Or, [Cho] analyzed effect of changing Spin structure on your result.

<u>Theorem</u> [Cho] Using the standard integrable J, there are $n+1$ families of Maslov 2 discs.

ex: 

$T^1_{cl} \subset \mathbb{P}^1$;



In $\mathbb{P}^2$, using a "toric model": the polytope is a base of a torus fibration $\mathbb{P}^2 \to \Delta$.

We can compute explicitely the homology classes in $H_1(T^n)$ the boundaries of these discs live in: basically the images of $\tilde{\alpha}_1, \dots, \tilde{\alpha}_{n+1} \overset{\text{generators}}{\in} H_1(T^{n+1})$, projected to $H_1(T^n)$.

$\Rightarrow$ Given a rank 1 local system ∇ on T^n with holonomies (in $\mathbb{C}^*$) equal to $x_1, \dots, x_n, \dfrac{1}{x_1 \cdots x_n}$ on $\mathrm{im}(\tilde{\alpha}_1), \dots, \mathrm{im}(\tilde{\alpha}_{n+1})$.

$\Rightarrow m_0(L, \nabla) := x_1 + \dots + x_n + \dfrac{1}{x_1 \cdots x_n}$.

Rem: can think of $m_0(L,\nabla)$ as being part of a function (as ∇ varies)

$W : (\mathbb{C}^*)^n \to \mathbb{C} : \nabla \mapsto m_0(L, \nabla)$ "Maslov 2 disc superpotential"

space of rank 1 local systems on L.

Theorem: [Cho] There are exactly $(n+1)$ rank 1 local systems on L with $HF^*((L,\nabla),(L,\nabla)) \neq 0$. (They are $\simeq H^*(T^n)$; the others are 0)

In fact, there is a bijection between $\{\nabla \mid HF^*(L,\nabla) \neq 0\}$ and $\mathrm{crit}(W)$.

$\leadsto \partial_i W$ (partial derivatives) have something to say about μ^1.

And, we can compute μ^2, etc, via higher partial derivatives of W.

This is a special partly worked out case of HMS:

$$\mathcal{F}uk(\mathbb{P}^n) \longleftrightarrow MF\left((\mathbb{C}^*)^n, W = z_1 + \dots + z_n + \frac{1}{z_1 \cdots z_n}\right)$$

"matrix factorizations"

$(T^n_{cl}, \nabla_i) \longleftrightarrow \mathcal{O}_{p_i}$ structure sheaf at a critical point

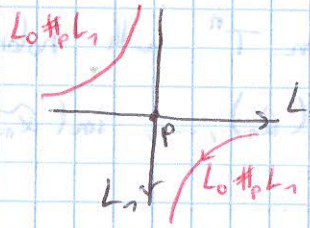
It will turn out that $\coprod_{i=1}^{n+1} (T^n_{cl}, \nabla_i)$ (split-)generate the monotone Fukaya category of $\mathbb{P}^n$.

We want to say what it means for $\{L_i\}$ to (split-)generate another $K \subseteq (X^{2n}, \omega)$.

Exact triangles.

We want to talk about long exact sequences in Floer homology.

ex: if $L_0 \pitchfork L_1 = p$ with $\operatorname{ind}(p) = 0$ (wrt some gradings on L_i),



we can define $L_0 \#_p L_1$ the "Lagrangian connected sum", or "Polterovich surgery".

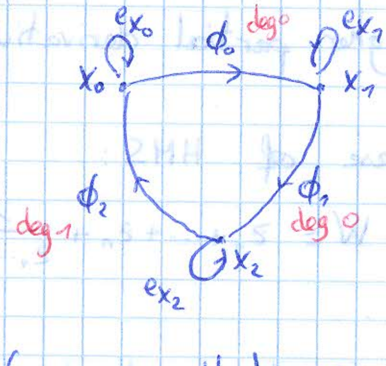
A result of Fooo's imply that for any Lagrangian K, there is a LES "natural wrt changing K"

$$\begin{tikzcd} HF^*(K, L_0) \arrow[rr, "{[\mu^2(-,p)]}"] & & HF^*(K, L_1) \arrow[dl] \\ & HF^*(K, L_0 \#_p L_1) \arrow[ul, "{[1]}"] & \end{tikzcd}$$

This is a shadow of: "$L_0 \#_p L_1$ is isomorphic to the mapping cone $L_0 \xrightarrow{p} L_1$".

One say that "there exists a triangle
$$\begin{tikzcd} L_0 \arrow[rr] & & L_1 \arrow[dl] \\ & L_0 \#_p L_1 \arrow[ul, "{[1]}"] & \end{tikzcd}$$
".

Abstract triangle category: let T_3 denote the (strictly unital) category:



, meaning: $\hom(X_0, X_0) = \mathbb{k}\langle e_{X_0}\rangle^{\deg 0}$

$\hom(X_0, X_1) = \mathbb{k}\langle \phi_0\rangle^{\deg 0}$

$\hom(X_1, X_0) = 0$

etc

We require that e_{X_i} are strict units, and

$$\mu^2(\phi_{i+1 \bmod 3}, \phi_i) = 0 \qquad \mu^3(\phi_1, \phi_0, \phi_2) = e_{X_2}$$
$$\mu^3(\phi_2, \phi_1, \phi_0) = e_{X_0} \qquad \mu^3(\phi_0, \phi_2, \phi_1) = e_{X_1}.$$

No μ^1, μ^2 and μ^3 as above, all higher μ^k's are 0.

Definition: an <u>exact triangle</u> in an A_∞-category $\mathcal{E}$ is (the image of) a functor $\mathcal{F}: T_3 \to \mathcal{E}$.

Rem: a nice aspect of this definition is that if $\mathcal{G}: \mathcal{E} \to \mathcal{D}$ is any A_∞-functor and $\mathcal{F}: T_3 \to \mathcal{E}$, then $\mathcal{G} \circ \mathcal{F}: T_3 \to \mathcal{D}$ is an exact triangle.

Rem: we can pull-back triangles up to quasi-isos along fully faithful functors.

$\leadsto$ $\mathcal{F}$ is (roughly) the data of
- * objects X, Y, Z in $\mathcal{E}$
- * closed morphisms c_0, c_1, c_2 in $hom^0(X,Y)$, $hom^0(Y,Z)$, $hom^1(Z,X)$

such that * $\mu^2(c_1, c_0) = 0$, or more generally is chain homotopic to 0, by a explicit homotopy $\mathcal{F}^2(\phi_1, \phi_0)$; we need to keep track of these. (Come from A_∞-functor relations)

* Massey products between c_2, c_1, c_0 (and all cyclic permutations), ~~equal to~~ whose difference from e_X is exact, with primitive involving $\mathcal{F}^2$'s, μ^2's and $\mathcal{F}^3$'s.

$\mu^3(\mathcal{F}^1(\phi_2), \mathcal{F}^1(\phi_1), \mathcal{F}^1(\phi_0))$

In a given $\mathcal{E}$, it's not true that every closed morphism $X \xrightarrow{\mathcal{E}} Y$ extends to a triangle.

Definition: an A_∞-category is <u>pre-triangulated</u> if
- * every closed morphism extends to a triangle
- * there exists shifts: given any $K \in ob\,\mathcal{E}$, $\exists\, K[1]$ with $\forall X, \forall K$,

  $hom^{\bullet}_{\mathcal{E}}(X, K[1]) = hom^{\bullet}_{\mathcal{E}}(X,K)[1] = hom^{\bullet-1}_{\mathcal{E}}(X,K) = hom^{\bullet}_{\mathcal{E}}(X[-1], K)$.

  The map $(-)[1]: \mathcal{E} \to \mathcal{E}$ is invertible, so $\exists\, X[n]$ $\forall n \in \mathbb{Z}$, and it is compatible with μ.

<u>Next time</u>: show that any $\mathcal{E}$ has a "pretriangulated hull" and also a "{idempotent / split-closed} triangulated hull" (unique up to quasi-iso) + explore applications to generations.

And there is a fully faithful embedding $\mathcal{E} \hookrightarrow \mathcal{E}^{\Delta} \hookrightarrow \underbrace{\mathcal{E}^{\Delta,\pi}}_{\text{idempotent pre-}\Delta\text{ hull.}}$