M. Abouzaid - Talk 2

* **The $\dim_{\mathbb{R}} = 2$ case:**

X open Riemann surface

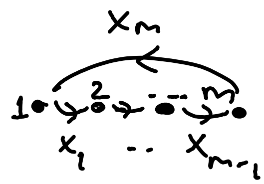
Q ribbon graph giving a skeleton of X

(ie. graph + cyclic ordering of edges adjacent to each vertex)

$\rightsquigarrow$ $\mathcal{B}$ cosheaf on Q:

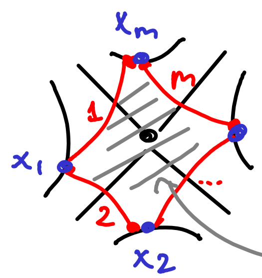
→ stalk at a smooth point $=$ $\bullet$ category with one object, morphisms $\mathbb{k}\cdot id$.

→ at a vertex of valence m $=$



$\mu_m(x_1, \dots, x_m) = id$

$\Longleftrightarrow$



the disc giving μ_m.

(End of each object $= \mathbb{k}\cdot id$
All other morphisms $=$ arrows).

The criterion of yesterday's talk holds

$$H_*(Q, HH_*(\mathcal{B})) \longrightarrow H_*(X, \partial X)$$

$$[Q] \longmapsto [X]$$

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* **Higher dimensions:** **first example**

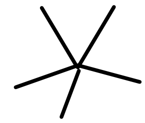
Q compact smooth manifold, choose a triangulation of Q

→ get $\mathcal{B}$ = constant cosheaf whose stalk is $\bullet \overset{\circlearrowleft}{}_{id}$ $(= HF^*(T^*_q Q, T^*_q Q))$ in "unwrapped" (Nadler-Zaslow) sense.

generation criterion holds.

This gives yet another proof that

$$\mathrm{Fuk}_{\mathrm{compact}}(T^*Q) \hookrightarrow \mathrm{mod}\text{-}C^*(Q) \quad (\text{assuming } \pi_1 Q = 0)$$

★ <u>Next example</u>: Q locally modelled on  $\times$ $(n-1)$-dim'l smooth mfld

ie. Q_i compact smooth mflds w/ boundary

P compact smooth $(n-1)$-mfld (not necess. connected)

$\partial Q_i \to P$ diffeo on each component

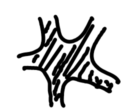
$Q = \left(\bigcup Q_i\right)/\sim$ $\quad\sim$ identifies pts of ∂Q_i with same image in P.

Choose also a locally constant cyclic ordering of preimages of pts of P.

$\Rightarrow$ cosheaf of A_∞-cat's on Q

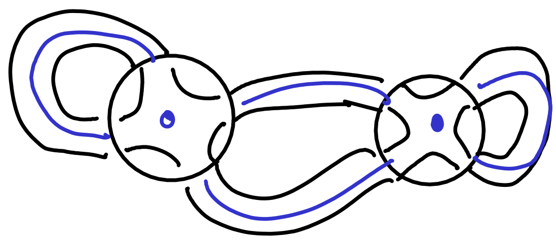
Again stalks $=$ $\bullet\circlearrowleft^{id}$ at smooth pts,  at singular pts.

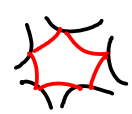
$\mu^k = id$

This is a skeleton for $X = \left(\left(\bigcup_i T^*Q_i\right) \cup \{T^*P \times$  $\}\right)/\sim$

valency depends on valency at component of P

("boundary plumbing" of T^*Q_i's at ∂Q_i).

<u>Ex</u>: $P = 2$ pts, $Q_i =$ intervals, 

At $p \in P$, the Lagrangians we take are $T_p^*P \times$ 

By projecting 'locally' we compute the stalk at $p \in P$ to be as above & see A_∞-structure is as in the 1d case.

This category satisfies the generation criterion of yesterday's talk

$\Rightarrow$ can compute $HF^*(K,K)$ from the associated modules.

<u>Goal</u>: understand how <u>usual plumbings</u> can be understood in terms of this picture.

Setup: N_1, N_2 closed smooth mflds

$B \hookrightarrow N_1$, $B \hookrightarrow N_2$ embeddings with isomorphic normal bundles $\simeq V$

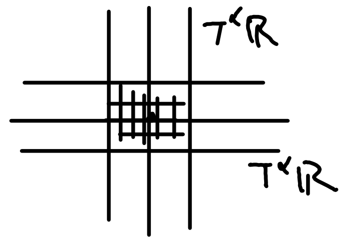
$V_{\mathbb{C}} \times_B T^*B$ ($\cong$ nbd of B inside T^*N_i)

carries an involution exchanging factors in $V_{\mathbb{C}} \simeq V \oplus V$

$\Rightarrow$ plumbing $:=$ glue T^*N_1 to T^*N_2 along $V_{\mathbb{C}} \times_B T^*B$

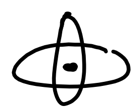
$=: T^*N_1 \mathop{\#\!\!\#}_B T^*N_2$

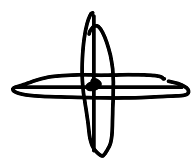
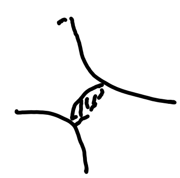
Ex: $pt \hookrightarrow \mathbb{R}$, $pt \hookrightarrow \mathbb{R}$ $\Rightarrow$



For codim $B = 1$ this clearly looks like above (make 4-valent vertices along B)

Problem: If codim $B > 1$, then there's no nice family of Lags in $T^*N_1 \mathop{\#\!\!\#}_B T^*N_2$ parametrized by $N_1 \cup_B N_2$.

Ex: $N_1 = N_2 = D^2 \supset B = $ origin  ?

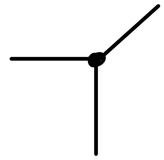
Idea (Kontsevich?): Replace  by 

i.e. plumbing has a skeleton with 3 "smooth components"

$\{N_1 - \text{open nbd of } B\}$ —— $\{N_2 - \text{open nbd of } B\}$

cyclic ordering

$\{$ disc bundle of normal bundle $V \downarrow B$ $\}$
($\Leftrightarrow$ nbd of B in N_i)

glued around  $\times$ (mfld)

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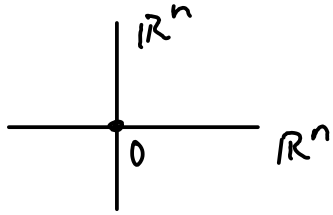
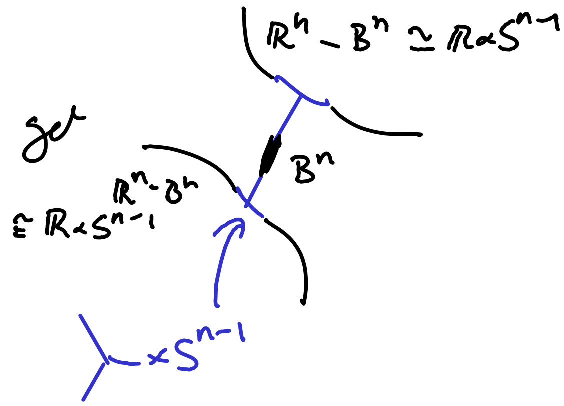
boundary of nbd of B in N_i

- Ie locally for 

, get 

Appl: Thm (A-Smith)

$N_1 = N_2 = S^n$, $B = pt$, $X = T^*S^n \underset{B}{\#} T^*S^n$

$= A_2$- ALE space 

$\Rightarrow$ in X, all exact Lags of μ index $= 0$ are Fukaya-isomorphic to an iterated Dehn twist of N_1. (around N_1, N_2) In particular they're all homology spheres.