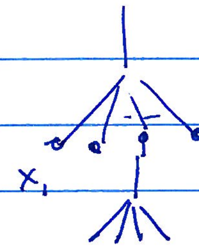
Tamarkin II

Monoidal category $\mathcal{M}$.

$X_1, \ldots, X_n$ $\quad \mathrm{Ob}\, \mathcal{M}$.

$X \quad \hom(X_1 \otimes \cdots \otimes X_n, X)$



colored asymmetric operad.

$\forall X \in \mathrm{Ob}\, \mathcal{M} = \mathcal{M}_1 \quad \mathcal{M}_0$

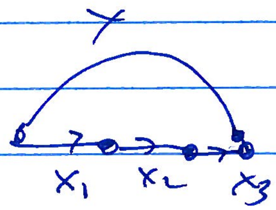
2-category:

$s(x) \xrightarrow{x \in \mathrm{Ob}\, \mathcal{M}} t(x)$

$\mathcal{M}_1 \underset{s}{\overset{t}{\rightrightarrows}} \mathcal{M}_0$.

set $\qquad$ set

$\mathcal{M}_0. \qquad \mathcal{M}_0$



$\hom(X_1 \otimes X_2 \otimes X_3, X)$

If one has a monoidal category $\mathcal{M}$ & $\mathcal{C}$, can define $\mathcal{M} \curvearrowright \mathcal{C}$. right action (have left action, etc.

$\mathcal{M} \otimes \mathcal{C}_L \to \mathcal{C}_L$

$\mathcal{M} \otimes \mathcal{C}_L \to \mathcal{C}_L$

if. $\mathcal{C}_R \otimes \mathcal{M} \to \mathcal{C}_R$

have notion of an algebra object in a monoidal category. just map Ass. alg operad to multioperation in monoidal category.

So if $A \in Alg(\mathcal{M})$

$\Rightarrow \mathcal{C}^{A\text{-mod}}$, category of A-modules in $\mathcal{C}$.

Can also have bimodules, & tensor product of modules!

$\mathcal{C}_{\mathcal{C}} \otimes_{M} \mathcal{C}_{L} \rightsquigarrow$ a category!

N is a monoidal category over N.

${}_M\mathcal{C}_M$, then get ${}_M N_M \otimes_{M} {}_M N_M \to N$

$M \to N$.

If $A \in Alg\, M$, then

$N^{A\text{-bimod}}$: will form a monoidal category,

objects are A w/ structure $A \otimes \cdots \otimes A \otimes X \otimes \cdots \otimes A \to X$

Now:

M is a closed sympl. manifold,

$[\omega] \in H^2(M;\mathbb{Z})$, $\omega = c_1(\mathcal{L})$, $\mathcal{L} \xrightarrow{f} M$.

$p^*\omega = d\theta$ (θ connection on bundle)

$\mathcal{L}$ is a circle bundle

How to produce symplectic ball?

Fix pseudo-kähler metric, unitary frames ~~gate~~ on M ~~[illegible]~~

$\times$ symps. ball of ~~[illegible]~~ small radius $\longrightarrow M$.

$$\mathcal{F}r_M \times B_R \longrightarrow M$$
$$\uparrow$$

Upgrade:

$$\underbrace{\mathcal{F}r_M \times_{M} \mathcal{L}}_{\mathcal{F}} \times B_R \longrightarrow \mathcal{L}$$

$$F \times B_R \xrightarrow{f} \mathcal{L}.$$

$$p^{-1}\theta_{\mathcal{L}}\Big|_{f \times B_R} = \alpha\Big|_{f \times B_R}.$$

Think of the following sheaf:

$$\mathcal{D}_{>0}(F \times \mathbb{R}^n \times \mathbb{R})$$
$$\cdot\, \mathrm{Cone}(T^*F \times B_R)$$

$$\mathcal{D}''(T^*F \times B_R)$$

<u>Goal</u>: find $A \in \mathcal{D}(T^*F \times T^*F\ \cancel{\times}\ B_R \times B_R)$

$\mathcal{M}$ (possibly curved) algebra/module on it?

Firstly, f ~~*~~ : $U(n) \times S^1$ acts on F

Lift

$$\tilde{U}(n) \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathcal{D}_{>0}(\widetilde{Sp(2n)} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R})$$

$$Q|_{\tilde{U}(n)} \longleftarrow\!\!\shortmid Q.$$

extend to a sheaf on

$Q' \in \mathcal{D}_{>0}(\tilde{U}(n) \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R})$ (add an action of $\mathbb{R}$ by shifting along time)

finally, have $\Big\downarrow \pi$

$$\mathcal{G} := \pi_* Q' \in \mathcal{D}^{(Alg)}_{>0}(U(n) \times S^1 \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}).$$

Alg: object b/c reflects action of group

$\Delta_{14}\times\Delta_{32}$

$\Delta_{12}\quad\Delta_{34}$

$X\ \ -X\quad X\ \ -Y$

$x\ \ x\quad y\ \ Y$

$\downarrow$

$\phi_t x\ \ \phi_{-t}x\ \ \phi_t Y\ \ \phi_{-t}Y$

$\Delta_{14}\quad\Delta_{32}$

$\phi_t x = \phi_{-t}Y$

$\phi_{-t}x = \phi_t Y$

$x \xrightarrow{1} \phi_t x = \phi_{-t}Y \xrightarrow{2} Y \xrightarrow{3} \phi_t Y = \phi_{-t}x \xrightarrow{4} x.$

$\mathcal{D}(F\times\mathbb{R}^n\times\mathbb{R})$ ↺

$U(n)\times S^1$, & $\mathbb{R}^n\times\mathbb{R}^n$ acts on $\mathbb{R}^n$.

so get

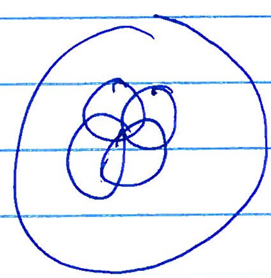
$\mathcal{M}^{\natural}$-mod. ~~flat module~~

and moreover have

$M^{\natural}\ \mathcal{D}(F\times F\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R})\ M^{\natural}$ (act on either factor on either side)

$\downarrow$

$\mathcal{A}$ an algebra



More formally:

$R \gg r$

$\mathcal{F}_{r,R} \subset F\times F\times M.$

$\mathcal{J}: F\times B_R \to \mathcal{L} \to M,$ image.

for $f\in F$: $B_R(f) := \mathcal{J}(f\times B_R)$

only depends on image of F in M here:

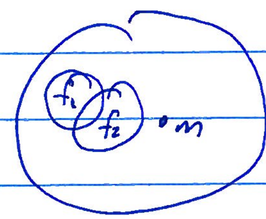
$F\times 0 \to M.$

For $m\in M \Rightarrow B_R(m)$

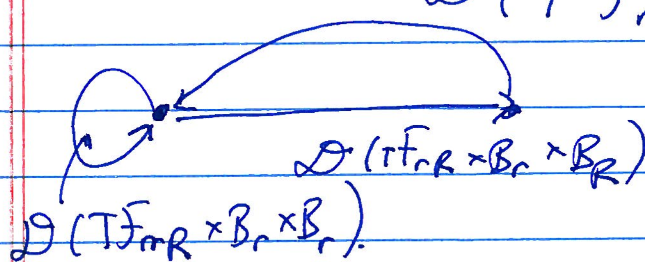
Claim: If choose two different frames, balls should differ by rotation! (?)

Then define $\mathcal{F}_{rr\,m} := (f_1, f_2, m) \mid B_r(f_1),\ B_r(f_2) \subset B_R(m)\}$

i.e.



$\mathcal{F}_{rR} \subset \mathcal{F} \times \mathcal{F}$

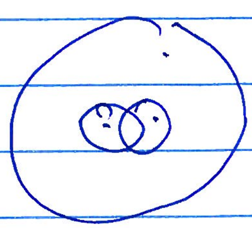


2-category: 0 cells are two points

1 cells are the categories above

all bimodules over $\mathcal{B}$.

idea: assign complexes to a picture



Then, find a way to glue them together, look at all possible configurations:

Def: $\mathcal{F}_{\underbrace{r \ldots r}_{n}, R} = \{ f_1, \ldots, f_n, \underset{M}{\hat{m}} \mid B_r(f_i) \in B_R(m) \}$

$$\begin{array}{ccc} & \mathcal{F}_{r\ldots r,R} & \\ \swarrow & & \searrow \\ (\mathcal{F}_{rrR})^{n} & & \mathcal{F}_{rrR} \end{array}$$

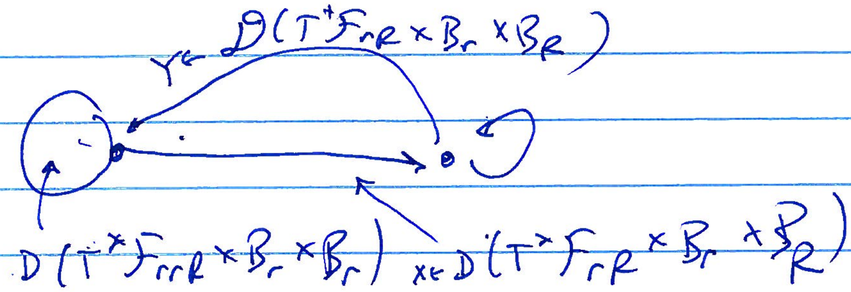
Have a functor $\mathcal{D}(T^*\mathcal{F}_{rrR} \times B_r \times B_r)^n \xrightarrow{\phi_n} \mathcal{D}(T^*\mathcal{F}_{rrR} \times B_r \times B_r)$

multi-hom

$$\hom(X_1 \otimes \cdots \otimes X_n, X) = \hom(\phi_n(X_1, \ldots, X_n), X)$$

tensor product representable, not strict.

Idea: as soon as you have objects X, Y



$\mathcal{M}$

$$X : pt \underset{Y}{\overset{X}{\rightleftarrows}} \mathcal{M}.$$

$$A = X \otimes Y \in \mathcal{M}.$$

& $Y \otimes X \to Id_{pt}$, $\Longrightarrow$ $X \otimes Y$ is an alg. in $\mathcal{M}$, s/c

$$X \otimes \underbrace{Y \otimes X}_{\downarrow \; Id.} \otimes Y$$

Luckily, we already had such X, Y from last lecture: How? Have family of sympl. embeddings,

$$F_{rR} \times B_r \to B_R \to T^*\mathbb{R}^n \qquad \text{this is } X.$$

Y same but transposed.

Have these for trivial reasons.

But this was just some fake category.

Have a projector $\mathcal{F}_{rrR} \xrightarrow{\pi} \mathcal{F}\times\mathcal{F}$ pair of disks w/ additional sh.

get functor

$$\pi^{-1}: \mathcal{D}_{>0}(\mathcal{F}\mathbb{R}^n \mathcal{F}\mathbb{R}^n \mathbb{R})$$
$$\downarrow$$
$$\mathcal{D}(\mathcal{F}_{rrR}\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R})$$

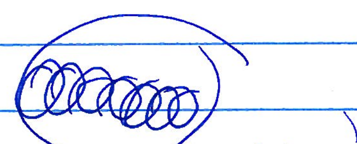
claim: object $X\circ Y \cong p^{-1}$ independent of ambient disc. $p: \mathcal{F}_{rrR}\to\mathcal{F}^2$. inverse image.

assign sheaf constant along ambient disc.

get monoidal functor, not always iso.

Consider first those sheaves supported on overlapping discs.

n-fold combos, n small:



either lie in big disk:

or composition 0 b/c of some gap.

gives map of operad, iso for small enough arity depending on $r<R$,

want isos for arity ≤ 3 at least.

How to extend any further?

T M.

$H^*(\text{Full operad}(\mathcal{A})) = H\mathcal{O}$. On derived level, get honest algebra $A\otimes A\to A$

Claim: suppose can find splitting eg

$A\to A\otimes A\to A$ on level of A-bimodules in $H(\mathcal{O})$

Claim: then can lift to algebra in $\mathcal{O}$ (not $H\mathcal{O}$).

why? can look at Hochschild complex, find homotopy. Problem: doesn't exist in our setting (need quasi-classical limit).

Now, think of our category as enriched over Λ, divide by Λ_{max}.

Given any such category $\mathcal{D}_{>0}(X\times\mathbb{R})$, define

$$\mathcal{D}_{>0}(X\times\mathbb{R})^{\varepsilon\text{-classical}} \quad \varepsilon>0, \text{ where}$$

$$\hom_{\varepsilon\text{-cl}}(X;Y) = \mathrm{Cone}\left(\hom(X;T_\varepsilon Y)\to \hom(X,Y)\right).$$

<u>Next part</u>: given algebra here, how to get rid of the epsilon.

Some argument by means of which we can lift to category w/o epsilons.

<u>Q</u>: For cotangent bundle, category w/ this procedure is a twist by $w_2(X)$, (by some signs)?

(apparently $\varepsilon\to\infty$ is classical)

(want Hochschild complex + true ~~BV~~ structure)

there is torsion, e.g. if $\varepsilon \geq$ displacement energy

$HF(L,L,\Lambda_{big})\neq 0$, but

$HF(L,L,\Lambda_{field})=0$.