Tamarkin III: keeps extra parameter w/ forms (T^*P).

Firstly, have $\mathcal{D}(T^*F \times B_r \times T^*F \times B_r)$ monoidal category under convolution.

$\overset{\cup}{A}$

Full op (A) (homs from all powers of $A \to A$)

In $\mathcal{D}_{>0}(-)$

can define homs between any two objects as a module over Λ nov.

i.e. filtered $\hom(X, T_c Y)$

all shifts

$T^*F \times B_r$ — M

L ← Lag'n corresp.

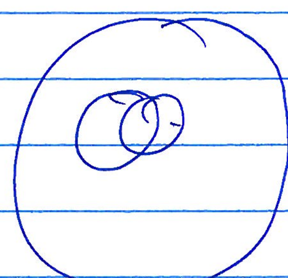
Each pt

nhood t. pass.

If two balls disjoint, stalk of A at $f_1, f_2 = 0$

if not, both balls in

ambient ball



get

transfer fcns: $\mathcal{D}(B) \xrightarrow{I_A} \mathcal{D}_{>0}(\mathbb{R}^n \times \mathbb{R})$ ⇝ assoc'd kernel A.

$\downarrow pr_2$ (problems w/ Maslov index, grading)

$\mathcal{D}(B)$

(problems w/ this convolution transition fn. kernel of at C-Y, etc)

what's the Lag. correspondence

$L_A \subseteq T^*F \times B_R \times M$, ? (get transition w/ composing by this)

have $F \times B_R \to M$.

graph $\Gamma \subseteq F \times B_R \times M$.

want a section of $T^*F \times B_R \times M$ over graph.

What's the cotangent vector associated to a point?

$\Rightarrow \omega \in T_p^*F$.

i.e. to a tangent vector, find a number.

↑

ham. vector field $\longrightarrow$ find ham. fxn.

b/c F is a smooth family of embeddings.

(modulo top. obstructions)

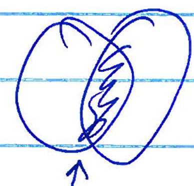
what's this over

$D(T^*F \times B_r \times T^*F \times B_r)$

↑ really means

$\mathcal{D}_{\cdot,0}(F \times \underbrace{R^n \times F \times R^n \times R}_{})$

open subset in cotangent bundle, localize. etc. etc.

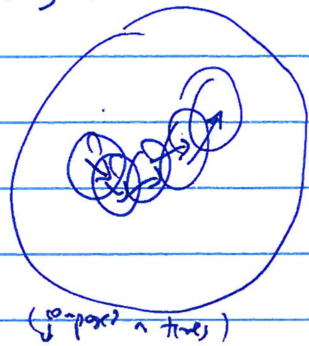


diagonal on overlap

want to make into an algebra on classical level.

alg. structure allows composition.

Only if arrows all within big ball:



(composed n times)

$$\mathrm{RHom}(A^{\otimes n}, T_c A) = \mathcal{O}_c(n).$$

all possible shifts.

Answer: graded space by real numbers, $c \in \mathbb{R}$

If $c_1 < c_2$, one mapped to another

Think of as module over Λ_{nov}, Gr(q^c part) is c above.

$q^c : \mathcal{O}_d \to \mathcal{O}_{d+c}$. the translation map.

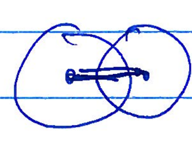
Now quotient by q^{ε}! $\varepsilon \to 0$

classical $\mathcal{O}_c^{cl} := \mathrm{Cone}\left(\varinjlim_{c_1 < c} \mathcal{O}_{c_1} \to \mathcal{O}_c\right)$

(corresponds to energy bds. for hol. disks).

Assoc $\longrightarrow \mathcal{O}_0^{cl}$ ← sits in 0th grading b/c want multiplication to have 0th grading

Have $A \otimes A \to A$



$\mathcal{L}$

periodicity:

$T_1 : A \xrightarrow{\sim} A$. $\underline{1}$ is power of sympl. form (periods should be multiples of 2).

so can consider a graded version

Assoc^{gr} who's n-ary operations

$$\mathrm{Assoc}(n)^{gr}_{c\in\mathbb{Z}} = k$$

$$= 0 \text{ if } c \notin \mathbb{Z}.$$

$\boxed{\text{Not really assoc., } A_\infty.}$

also have iso.

$$A \to T_2 A.$$

To get $\mathrm{Assoc} \to \mathcal{O}''$, first construct binary & ternary operations, & use splitting

$$A \to A \otimes A \to A \text{ to}$$

get homotopies.

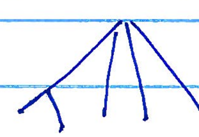
Rmk: we were never strict, get A_∞ structure, not associative.

so have: $\mathrm{Assoc} \to \mathrm{Assoc}^{gr} \to \mathcal{O}$

Def: Call $\mathrm{Assoc} \xrightarrow{f} \mathcal{E}$ nice if

$m_2 \in \mathrm{Assoc}_2$: $f(m_2) : \mathcal{E}(n) \xrightarrow{\sim} \mathcal{E}(n+1)$

(really, mean A_∞)



~~the structure~~ of

The category of nice guys is equiv to category of → corresp. gerst algebra Gerstenhaber algebras ??

$\mathrm{Assoc} \longrightarrow \mathcal{O}$ is Hochschild complex

$$\begin{tikzcd} \mathrm{Assoc} \arrow[rr] \arrow[dr] & & \mathcal{O} \\ & \mathcal{E} \arrow[ur] & \end{tikzcd}$$

If have a map

$$\mathrm{Assoc} \to \mathcal{O}$$, can

construct a deformation complex

$$\mathrm{Def}(f) = C^{\bullet}(\mathcal{O}, \mathcal{O})$$

deformation complex

differential given by bracketing with f.

(tensor w/ artinian ring, get A-C elts.)

Generalize this to

$$\mathcal{E} \xrightarrow{f} \mathcal{O} \quad \underline{\text{nice}} \text{ case}$$

$$\mathrm{Assoc}^{gr} \xrightarrow[f]{} \mathcal{O}^{cl} \xrightarrow{id} \mathcal{O}^{cl}$$

def (id) is a monoid, by below

~~Defn~~

(general situation:

$$\mathcal{E}_1 \xrightarrow{f} \mathcal{E}_2 \xrightarrow{g} \mathcal{E}_3$$

get $\mathrm{def}\, f \otimes \mathrm{def}\, g \to \mathrm{def}(gf)$

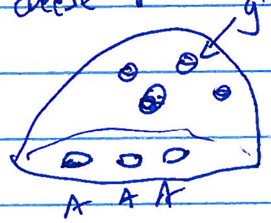
tensor product).

$$\mathrm{Assoc}^{gr} \xrightarrow{f} \mathcal{O}^{cl} \xrightarrow{id} \mathcal{O}^{cl}$$

def(f). $\quad$ g

L $\quad$ 3-algebra.

Mixture given by 3-D swiss cheese operad



gives a module operations to A.

Any $\mathbb{Z}$-algebra is a Lie algebra,

so get a M-C elt.

$\sum X q^m$

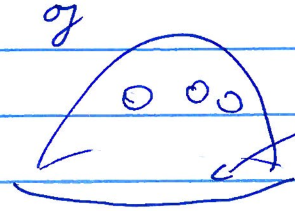
ϕ ↖ total grading should be zero, all corrections higher order.

nothing zero graded.

Cb/c no zero area instanton corrections

L module over $\mathfrak{g}$

M.C. elt in $\mathfrak{g}$

Curvature:  no elts. here, live in A

obstruction.

L $\mathfrak{g}$

If have M-C. elt. in $\mathfrak{g}$,

should get new thing of same type as L over

Novikov ring.

May get curved gerstenhaber algebra

Really don't want $\mathfrak{g}$,

want

L $\mathfrak{g}$

$\cup$ e.g. in $CC^*(A,A)$

$\mathfrak{g}_{>0}$ $A \oplus hom(A,A) \oplus hom(A^2,A)$ ——

↖ kill this one.

Corresponds to non-curved deformations of $\mathcal{C}$.

M. C. elt. $X \in \mathfrak{g}$

$\Big\downarrow \qquad \cup$

find some $\quad g^{\geq 1}$

from $\bar{X} \in$

$\bar{Q}$: find $\bar{X} \in \mathfrak{g}$ from M. C. elt. in g.

How? By using extra structure: circle acting on whole story.

How? cyclic trace..

$\cancel{A} \in \mathcal{D}(X \times X)$

$M_i \in \mathcal{D}(X \times X)$

$Tr(M_1, M_2, \ldots, M_n)$

$X^{M_1} X^{M_2} \cdots X \cdots X.$

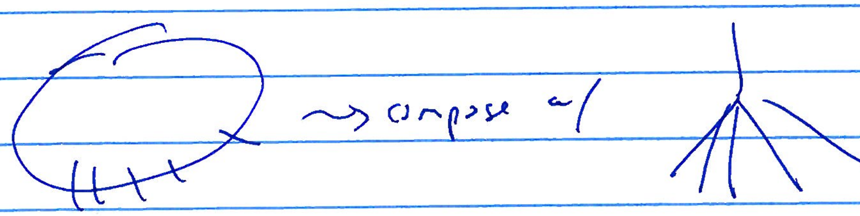
So can construct $Tr(A, \ldots, A) = \mathcal{O}^{\bullet}(n)$

$\cap$

$\mathcal{O}(n)$.

Can compose:

$$\mathcal{O}^{circ}(n) \otimes \mathcal{O}(m) \longrightarrow \mathcal{O}^{circ}(n+m-1)$$



~~Hochschild calculus structure~~

If: have $\Omega \in C_\bullet(A)$ actually is $HH_\bullet(A)$

$$C_\bullet(A) \otimes C^\bullet(A) \longrightarrow C_\bullet(A)$$

s.t. Ω gives isomorphism.

& operad nice.

then get BV operad.

Then this structure is a BV structure. (collapse $HH^\bullet$ and $HH_\bullet$)

Q: where's Ω??

~~$\mathcal{O}(A)$~~

Then:

$b \in (F \times_n F \times R^n \times R^n)$ $\exists$ $\mathcal{O}(b) \longrightarrow \mathcal{O}(A)$

premage of alg. which quantizes rotating.

Assoc

Furthermore, Hochschild complexes of these are the same

$\underline{C^\cdot(A,A)}$

$$A \underset{\mu}{\overset{\gamma}{\rightleftarrows}} A \otimes A.$$

~~HH(A)~~ $\mathfrak{h}$ is kernel of $A \circlearrowleft P$

Ω comes from completion of $HH^\cdot$ of $\underline{\mathcal{O}(\mathfrak{h})}$.

$\mathfrak{h}$ is Lie on cotangents of M which are equivariant, come to $\tilde{M}$.

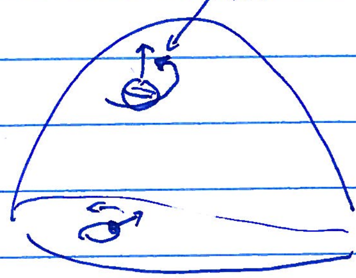
Now:

L def(f) is a BV algebra, ~~alg~~ upgrade $\mathfrak{g}$: a BV 3-algebra

swiss-cheese:

3-operad

vertical axis preserved



Furthermore, in the $\mathcal{O}^{cl}$, also have trace, so

X is circle invariant

(we're looking for deformations of circle operad)

So ~~get~~ look at cyclic complex $\mathfrak{g}[[t]]$.

Problem: this M-C. elt. lives in

$\mathfrak{g}$ rather than

$\mathfrak{g}_{\geq ?}$.

want to ~~see~~ upgrade to $\boxed{\mathfrak{g}_{\geq 1}}$.

this will imply a lift from

$$\begin{tikzcd}
 & \mathcal{O}^{cl} \\
\mathrm{Assoc} \arrow[ur] \arrow[r] & \mathcal{O} \arrow[u]
\end{tikzcd}$$

· Take equivariant cohomology: $\mathfrak{g}[[t]]$ w.r.t. circle action.

Condition which resolves this:

need to know how circle acts on L, G

~~Theorem~~

- suppose circle action on L can be trivialized.
- S^1 action on G "is nilpotent"

really: on $\mathfrak{g}[[t]]$

$t^n \sim 0$ on $\mathfrak{g}$.

Then: there is a lifting.

~~Good~~ news: need to know $\mathfrak{g}[[t]]$.

Get something like. $C_*(L^2 M)$. for $\mathfrak{g}$., all $S^2 \to M$, connected components labeled by ~~[~~ $\int_S^2 \omega$.

circle acts on obvious way. graded space

Need to prove S^1 acts nilpotently for non-zero degrees.

Equivariant cohomology, module over $k[[t]]$, $|t|=2$.

condition: t^n acts by 0, $n>>0$.

~~Nat(Id, Id)~~

~~So why is the action on $\mathcal{L}$ trivial?~~

why is S^1 on $\mathcal{L}$ trivial? immediate, computation.

can compute that def(f) is $H^\cdot(M)$, trivial S^1 action.

Then, prove criterion..

Sympl. form arose: standard balls,

M-C. elt.

final category is modules over

this curved algebra