

M compact symplectic

\leadsto dg category w/o using hol. disks (defined only using sheaf theory)

via

$T^*X \rightsquigarrow \text{sh}(X) \subset \text{shv}(X \times \mathbb{R})$

$\{F \in \text{shv}(X \times \mathbb{R}); \Gamma_c(U \times (-\infty, a], F) \sim 0\}$

$\forall U$ open, $\forall a$.

(can interpret an object in $\text{sh}(X)$ as a filtered sheaf on X w/ some condition.)

Assumptions

1) M is compact.

$[\omega]$ has integral periods $\Rightarrow \exists$

$S^1 \subset L \xrightarrow{\pi} M$ ~~over~~ $\int [\omega] = c_1(L)$.
(an 1.)

e.g. exists $\theta \in \Omega^1(L)$ S^1 -equivariant, δ

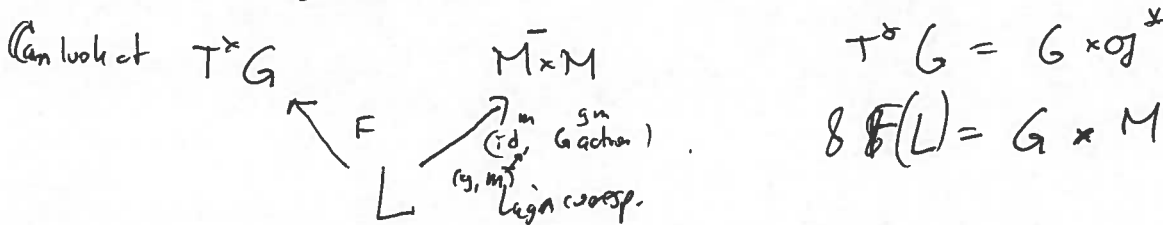
$\begin{cases} \theta|_{\frac{\partial}{\partial \psi}} = \mathbb{1}, \delta \\ d\theta = \pi^*\omega. \end{cases}$

θ turns L into a contact manifold G inf. dim'l Lie group

δ look at ~~contact equiv~~ $\text{Ham}(M) := \{F \in \text{Diffco}(L); F \text{ is } S^1\text{-equiv}, \delta F^*\theta = \theta\}$

(up to perturbation, any elt of this group is always covered by a ham. vector)

Have a map ~~\mathcal{O}^*~~ $M \subset \mathcal{O}^* =$ "functions on M "

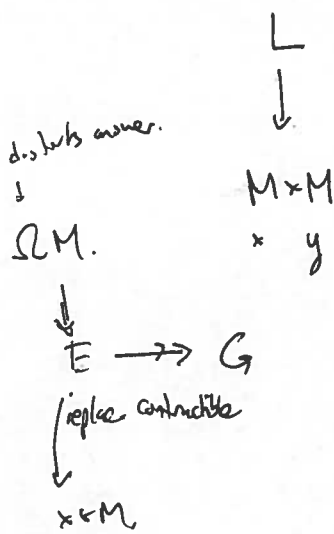


If ~~either~~ then, on the nose, $\bar{M} \times M$ gets realized as a ham. reduction of ~~T^*G~~ T^*G is isotropic in

Look at

$$\text{sh}(G) \supset \text{sh}(G)_{[F(L)]}$$

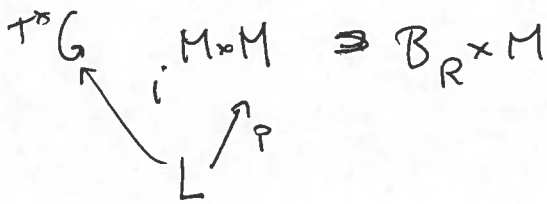
↑ singular support = $G \times \text{supp}(-)$.



has fiber $L_{(x,y)} = \{g \in G \mid gx=y\}$.

Undoubling:

Fix $B_R \subset M$.



$$\text{sh}(G.) [i p^{-1}(B_R \times M)]$$

want to undo this,
 b/c every sheet on B_R is
fusion; has finite capacity β
 is filled by it.

$$B_R \text{ sh}(G)_M$$

Assume $B_R \subset B'_R$.

Have a group $G_0 \subset G = g; \text{supp } g \subset B'_R!$

↑ regularly supported group.

Can do now

$$B_R \text{ sh}(G_0)_{B_R} \cong \text{sh}(\mathbb{R}^n, \mathbb{R}^n)_{(B_R \times B_R)}$$

$$\downarrow \text{act on}$$

$$\text{sh}(\mathbb{R}^n)_{(B_R)}$$

and also

ack

$$\text{Sh}(G)_{B_R} \hookrightarrow \text{Sh}(G)_M$$

have $\text{Sh}(G)_{B_R} \rightarrow \text{Sh}(G_0)_{B_R} \hookrightarrow \text{Sh}(B_R)$

$\mathcal{M} \hookrightarrow \text{Sh}(B_R) \oplus \text{Sh}(G)_{B_R}$
 find sth. action, need to undo
 not all objects are ~~locally~~ objects, so can use this category for proving non-displaceability
 have G equiv, so stable under G action by right.

(actually, replace G by $E =$ 'space of based paths' in G)

I want to "undo" G action.

can describe to see exact. $\mathcal{M} := \text{End}_{\text{Sh}(G)_{B_R}}(\text{Sh}(B_R))$

where $\mathcal{M}_{cl} \cong \text{Loc}(\Omega M)$ tw. \leftarrow twist/what twist? (Lurie, Jir-Threuman)
 group of coinvariants (nothing if work over 2-par. complexes)

can see that as global sections of formal sheaf on M ,

I need to take locally free sheaf on ΩM , but also shift along Narasimha

sheaf of spectra & compatible w/ G action;

really $\text{Pic}(S) \times \mathbb{R}$ shift along Narasimha

classifying data of sheaf is a map

spectra or actually Narasimha of spectra $B^2 \text{Pic}(S)$

$$M \xrightarrow{\text{develop}} B^2 \text{Pic}(S) \leftarrow \text{this only depends on } M \rightarrow B\mathbb{P}(2n) \leftarrow \text{J-hom, Botts etc.}$$

gives $\Omega M \rightarrow B \text{Pic}(S)$ group map: gives a monoidal category

Need to find M

Guess what M_{cl} is, solve deformation problem to lift \downarrow

$$M := \text{End}_{\mathbb{R}} \left(\begin{matrix} \text{sl}(B_{\mathbb{R}}) \\ \mathbb{R} \text{sh}(G) \\ \mathbb{R}^2 \end{matrix} \right)$$

What's M_{cl} ?

have alg in monoidal cat

M_{cl}

$\text{Loc}(\Omega M)_{tw}$
const. sheaf

& take $M_{cl} \text{ mod}$ (modules are alg in monoidal cat)

Now, lift to quantum level.

Ingredients:

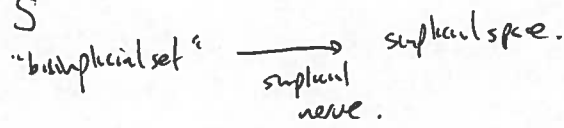
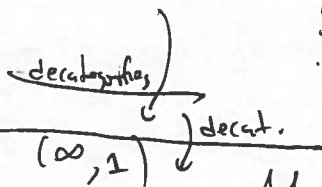
\mathcal{M} monoidal cat. (or more generally $(\infty, 2)$ cat).

"bimultiplicial set"

Then, $\int_{S^1} \mathcal{M}$ is $(\infty, 1)$.

$F \in \mathcal{M} \text{ mod.}$ dualizable
function (by $\text{inverts } 2\text{-morphisms}$)

$\text{ch}(F) \in \int_{S^1} \mathcal{M}$ object of cat.



\mathcal{M} usual dg cat. \rightarrow $C_0(\mathcal{M})$ $(\infty, 0)$ category.

\uparrow Hochschild chains of cat/alg

Now, if one has a dualizable module over \mathcal{M} ,

$$F \rightarrow \text{ch}(F) \in C_0(\mathcal{M})$$

$(\infty, 2)$: Get a functor

$$\mathcal{M}^x \xrightarrow{\text{ch}} \left(\int_{S^1} \mathcal{M} \right)^{S^1} [u^{-1}]$$

(underlying $(\infty, 1)$ category)

(downstairs, has an analogy, cyclic homology.

\downarrow Lurie

"periodic cyclic homology."

$\left(\int_{S^1} \mathcal{M} \right)^{S^1}$ nice advantage

Point: $\therefore (\int_{S^1} M) [u^{-1}]$ satisfies crystalline property / homotopy invariance;
 so objects in ~~\mathcal{M}~~ \mathcal{M}_{cl} are same as in ~~\mathcal{M}~~ \mathcal{M} .

$$\mathcal{M}^x \xrightarrow{ch} \left(\int_{S^1} M \right)^{S^1} [u^{-1}]$$

$$\uparrow \text{1/2}$$

$$ch(u_{cl}) \in \left(\int_{S^1} M_{cl} \right)^{S^1} [u^{-1}]$$

TR \subset \uparrow what are objects here? they're traces on M ; maps $M \rightarrow \text{Vect}$ invariant / in cycles basically
 \uparrow this map exists for free.

why does one need it? want to encode fact that \mathcal{QH}^* has trivial TR.

If define TR property, ~~$\mathcal{QH}^*(X)_{cl} = \text{hom}$~~

$$1) \text{hom}_{cl}(\text{TR}, ch(u)_{cl}) \cong C^*(M) \text{ w/ trivial circle action } (u).$$

$$2) \text{End}(\text{TR}) \otimes \mathbb{Q} \sim 0.$$

(so, can't get two different traces by this)

have a chosen fundamental class

$$\text{TR} \rightarrow u; \quad \text{this quantizes by crystalline property}$$

\uparrow quantum,

Wants: Find $v \in \mathcal{M}$ M -mod

$$\text{ch}(v) \xrightarrow{\sim} \text{ch}(u).$$

\uparrow TR

Why does this problem now have a solution?

Say have

$$F: X \longrightarrow Y \text{ over } \Delta$$

\downarrow
 P

$$X_0 \longrightarrow Y_0 \supset \{ F(p_0) \xrightarrow{\sim} p_0.$$

say, have classical retraction.

q_0
 \nwarrow classical pre-image

$$\Rightarrow \text{End}_{X_0}(q_0) \xrightarrow{\sim} \text{End}_{Y_0}(p_0)$$

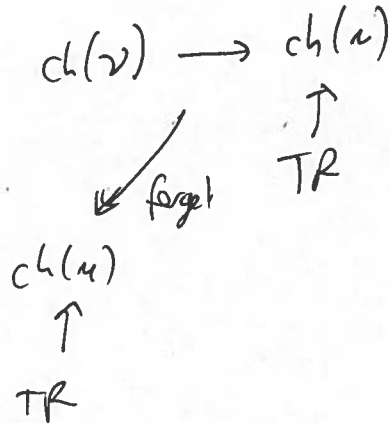
Then, there is a canonical way to lift

$$X_0 \rightsquigarrow X.$$

Then, define category in right way.

X will be all tuples

& Y will be



Calculate endomorphisms,
 it will not?