10/25/2016, D. Treumann, Stokes structures (cont'd from last time)

Last time:

$(\mathcal{E}, \nabla)$ vector bundle & connection on $\mathbb{P}^1 - \{0, \infty\}$.

regular at 0 & irregular at ∞.

$\rightsquigarrow$ (by looking at formal structure near ∞) "Levelt curve" $\subset$ $\hat{D}^*_\infty \times \mathbb{C}$, where $\hat{D}^*_\infty = \mathrm{Spec}\, \mathbb{C}((z^{-1}))$ is the formal disc around ∞.

$$\text{Levelt curve} = \coprod_a \text{graph of } p(z^{1/k_a})$$

some polynomials w/ no constant term.

Have

$$\begin{array}{ccc} \text{Levelt curve} & \overset{\text{"log boundary"}}{\rightsquigarrow} & \coprod_a S^1_a \\ \downarrow & & \downarrow \pi \\ \text{formal disc } \hat{D}^* & \underset{\text{"log boundary"}}{\rightsquigarrow} & S^1 \end{array}$$

with $S^1_a \to S^1$ $k_a : 1$.

Deligne defines ~~a ordering~~ a partial ordering on the fibers of π:

Given $\theta \in S^1$, & given $\theta_a \in S^1_a$ & $\theta_b' \in S^1_b$ preimages of θ, so $\pi(\theta_a) = \pi(\theta_b') = \theta$,

Say

$$\underline{\theta_b' < \theta_a} \quad \text{iff} \quad \left| \frac{\exp\big(p_b(\overbrace{r^{1/k_b} e^{i\theta_b'}}^{\text{"}z^{1/k_b}\text{"}})\big)}{\exp\big(p_a(\underbrace{r^{1/k_a} e^{i\theta_a}}_{\text{"}z^{1/k_a}\text{"}})\big)} \right| \to 0 \quad \text{as } r \to \infty$$

where r is the radial coordinate near ∞ on $\mathbb{P}^1 \setminus \{0, \infty\}$

$(\mathcal{E}^\nabla)_\theta$ = flat sections of $\mathcal{E}$ defined in a sector around ∞, of angle θ.

Deligne defined a filtration of $(E^\nabla)_\theta$: if $\theta_a \longmapsto \theta$,

Q: should this be $re^{i\theta}$? (or rather, $|s(re^{i\theta})|$?

then $$F_{<\theta_a}(E^\nabla)_\theta := \left\{ s \;\middle|\; \frac{\|s(r^{1/k_a} e^{i\theta_a})\|}{|\exp(p_a(r^{1/k_a} e^{i\theta_a}))|} = O(r^N) \text{ for some } N \right\}$$

Example: If $\nabla = d - \left(Y + \frac{F}{z}\right) dz$ where $Y, F \in M^{n\times n}(\mathbb{C})$

(regular at 0, irregular at ∞, but "not very"),

then "Levelt curve" has all $p_a = c_a z$ (Rmk: this may require Y semisimple?)

(Solutions all $\sim e^{c_a z}$)

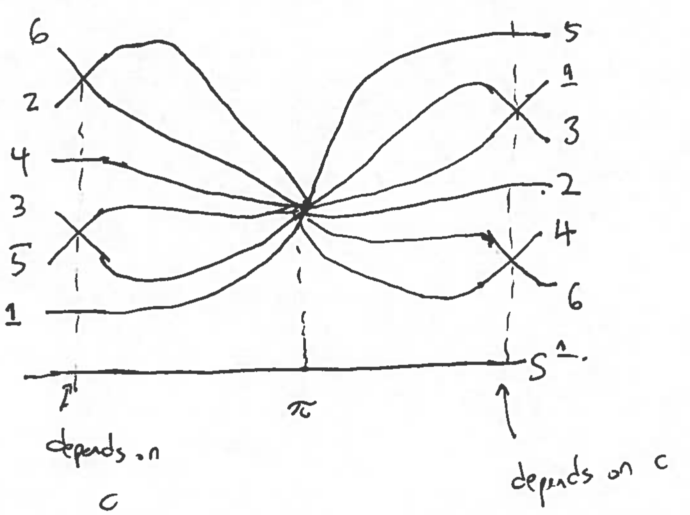
Example: $\exp(\sqrt{z} + c\sqrt[3]{z})$ obeys a 6th order ODE (which is quite complicated; coeffs. are rational w/ large numerators & denominators).

The associated levelt curve is

$$(z, \sqrt{z} + c\sqrt[3]{z}) \subset \hat{D}^* \times \mathbb{C}.$$

Here's an image of the "log boundary":

$$\coprod_a S'_a = S^1_a \xrightarrow{6:1} S^1$$



partial order by height.

(formally, this is irreducible;)

& formal splitting does not imply actual splitting.