11/15/2016, Mauricio Romo, Gamma Class from Gauged Linear Sigma models and B-brane transport

$\underbrace{\text{Gauged Linear Sigma models}}_{\text{GLSM}}$

GLSM & central charge of B-branes

- Motivation
- GLSM def. + examples
- B-brane transport on GLSM (+ Grade Restriction Rule (GRR))
- Physical partition functions ~~~> central charge
- Comparison with Iritani's formula (on geometric phases)

Motivation: X - kähler, compact, smooth (usually, CY).

$$g_{\mu\bar\nu} = \partial_\mu \bar\partial_{\bar\nu} K \leftarrow \text{kähler potential}$$

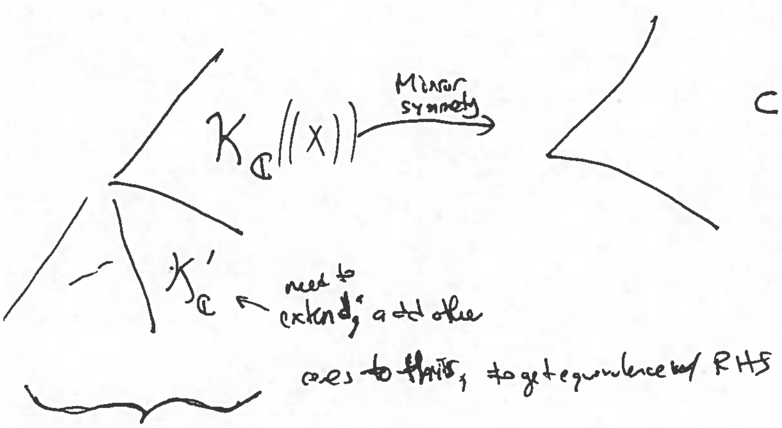
$$\rightsquigarrow \omega = i g_{\mu\bar\nu}\, dX^\mu \wedge d\bar X^{\bar\nu} \quad \text{kähler form} \in H^2(X;\mathbb{R}).$$

$\mathcal{K}_X :=$ Kähler cone $\subset H^2(X;\mathbb{R})$, defined by:

if $\dim_{\mathbb{C}} M_r = r$, then $\int_{M_r} \omega^r > 0$.

Also have complexified kähler cone

$$\mathcal{K}_{\mathbb{C}} := \underset{\uparrow \sqrt{-1}}{i}\mathcal{K}_X + H^2(X;\mathbb{R}/\mathbb{Z}) \ni \underset{\text{"B fields"}}{B} + i\frac{\omega}{2\pi}.$$



$\subset \mathcal{M}_{CS}(X^\vee)$ ← mirror (X) (but RHS is much larger generally)

$\mathcal{M}_K$ = stringy (quantum) kähler moduli of X.

GLSM (Witten '93):

Field theory that can cover all of $\mathcal{M}_K$:

("UV model" (not superconformal / no SC-symmetry))
but flows to a SCFT

GLSM : Input data: $V \cong \mathbb{C}^N$ cplx. vec space. superpotential: $W \in \mathrm{Sym}\, V^*$, G-invariant — all "$N=(2,2)$ SUSY theories in 2D"

$$\left(G,\ \rho_m : G \longrightarrow \underbrace{SL(V)}_{\text{CY condition: lands in } SL(V) \text{ instead of } GL(V)},\ W : V \longrightarrow \mathbb{C},\ t_k\ (k=1,\dots,s) \right)$$

compact Lie group — "Matter representation"

// for simplicity

$H \times U(1)^s$

product of simple Lie groups

$$t_k = \underset{\mathbb{R}}{\zeta_k} - i\,\underset{\mathbb{R}/2\pi\mathbb{Z}}{\theta_k}$$

Two classes of distinguished parameters:

- coeffs$(W) \sim$ "$\mathcal{M}_{cs}$" ("cplx-structure moduli"),
- $\{e^{t_k}\}_{k=1}^{s} \in \mathcal{M}_K$.

Rmk: there is also "R-symmetry" which is like a $U(1)$-action / grading on V:

$R : V \longrightarrow \mathbb{Q}$ (grading) (rat'l weights) s.t. $W(\lambda^R \cdot \phi) = \lambda^2 W(\phi)$ ("quasi-hom. weight 2")

Ex 1: Quintic : $\{f_5 = 0\} \subset \mathbb{P}^4$. (smooth)

$G = U(1)$, $V = \mathbb{C}^6$, ρ_m has weights $(-5, 1, \dots, 1)$

$(p, x_0, \dots, x_4)$.

$W = p \cdot f_5(x)$. $t = \zeta - i\theta$.

$\rightsquigarrow$ classical space of vacua

$$X_\zeta := \mathrm{Crit}(W) \subset V /\!\!/_{\zeta} G.$$

("a guide for what's in the infrared").

("just when $|\zeta| \gg 1$").

moment map of G-action $\mu = -5|p|^2 + \sum |x_i|^2$.

$$\mu^{-1}(\zeta)/G = \begin{cases} \mathcal{O}(-5) \to \mathbb{P}^4 & \zeta > 0 \\ \mathbb{C}^5/\mathbb{Z}_5 & \zeta < 0 \end{cases}$$

$\zeta > 0 \longrightarrow X_\zeta = $ quintic $\subset \mathbb{P}^4$. (in nice cases, trivial stabilizer on of $U(1)$ at any pt. found)

$\Rightarrow$ SCFT is NLSM w/ target quintic

non-linear sigma model

$\zeta < 0 \longrightarrow X_\zeta = [pt./\mathbb{Z}_5]$

$[\mathrm{Hess}(W)|_{X_\zeta}] \Rightarrow X_i$ fields are massless (p is massive)

says which fields acquire masses & which don't

$\rightsquigarrow$ (SCFT is an LG orbifold $((W(p=1), \mathbb{C}^5)/\mathbb{Z}_5)$.

Comment:

$\mathcal{M}_K$: of quintic is a cylinder

"large radius limit"

NLSM on $\{f_5 = 0\} \subset \mathbb{P}^4$

$\uparrow \zeta$

θ.

careful analysis reveals $t = 5\log(-5)$

discriminant conifold point "ζ" "twisted potential"

$\downarrow$

$-$ LG $\mathbb{Z}_5$-orbifold.

"Gepner point"

(Rmk: not at $\zeta = 0$ b/c cannot trust classical space at $\zeta = 0$).

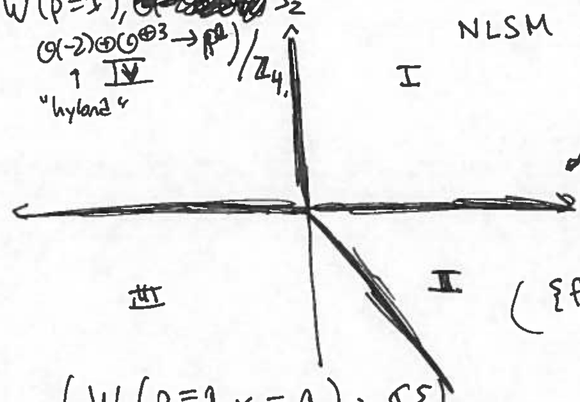
Ex. 2 (the octic)

$G = U(1)_1 \times U(1)_2$, $V = \mathbb{C}^7 \ni (p, x_1, \dots, x_6)$, & ρ_m has weights:

	p	x_1	x_2	x_3	x_4	x_5	x_6
$U(1)_1$	-4	0	0	1	1	1	1
$U(1)_2$	0	1	1	0	0	0	-2.

$$W = p\cdot f_{(4,0)}(x), \qquad t_j = \xi_j - i\theta_j$$

In this case, $\mathcal{M}_K$ is (or rather, its projection to (ξ_1, ξ_2) real part):



"large radius" NLSM $\{f=0\} \subset T$ ← toric, which is the resolution of

"secondary fan" (generalization of $\xi=0$ in prev. case?)

weighted projective space ~~orbifold~~,

$\left(\{f=0\} \subset \mathcal{O}(-8) \to W\mathbb{P}^4_{[1:1:2:2:2]}\right)/\mathbb{Z}_2$ ← has orbifold points

"NSLM w/ orbifold target"

Rmks: there are also discriminant points.

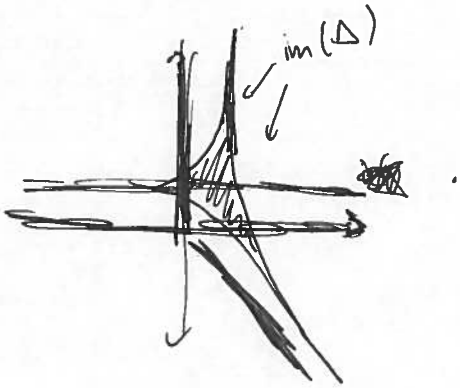
In general $\mathcal{M}_K = (\mathbb{C}^*)^s \setminus \Delta$ ← cplx. codim 1 closed set. ($\mathbb{C}$ compatible!)

In this case,

Δ: using $e^{-t_i} = q_i$:

$$\Delta = \begin{cases} 2^{18} q_1^2 \cdot q_2 - (1 - 2^8 q_1)^2 = 0 \\ q_2 = 2^{-2}. \end{cases}$$

projecting on $\mathbb{R}^2$:



Ex:3: Rødland model:

$G = U(2)$ $\qquad V = \mathbb{C}^7 \oplus (\mathbb{C}^2)^{\oplus 7}$

ρ_m: $(\det^{-1})^7 \oplus \square^{\oplus 7}$ ← 7 copies of fundamental.

The potential

$$W = P_a A^a_{ij} \varepsilon^{\alpha\beta} X^i_\alpha X^j_\beta$$

metric?

where: $i, j = 1, \dots, 7$

$\beta, \alpha = 1, 2$ and $\{A^a_{ij}\}_{a=1}^{7} \to 7\times 7$ anti-symmetric matrices

$a = 1, \dots, 7$ (take generic enough to ensure everything nice)

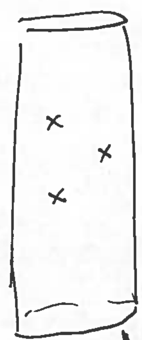
& only one kähler param (b/c only one $U(1)$ factor)

$t = \zeta - i\theta$.

7 hyperplanes in $G(2,7)$

NLSM on $G(2,7)[7]$

$\mathcal{M}_K =$



[Hori–Tong '06]: NLSM on Y

$$Y = \{ rk(A(p)) \leq 4 \} \subseteq \mathbb{P}^6.$$

contract A w/ p; ~~they~~ ~~stacky~~.

Rmk: these two are not birationally equivalent, unlike past examples!

On SCFT we can "define" B-branes; e.x. for $Z = \{f=0\} \subset T$,

it's $D^b coh(Z)$

Ex:

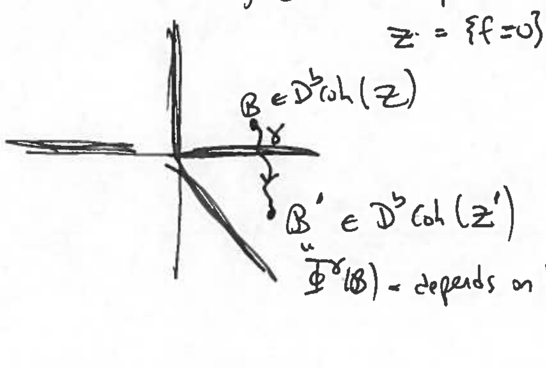
- on $(W(p\equiv 1, x_6\equiv 1); \mathbb{C}^5)/\mathbb{Z}_8$

it's $MF_{\mathbb{Z}_8}(W_{LG})$. & "global" (non-affine) version for $(W(p\equiv 1), \mathcal{O}(-2)\oplus \mathcal{O}^{\oplus 3} \to \mathbb{P}^1)/\mathbb{Z}_4$.

Since the picture for $\mathcal{M}_K$ involves moving around kähler (or CS) moduli space,

expect all B-brane categories in the picture to be equivalent:

$Z = \{f=0\} \subset T$.



- Determining Φ^{γ} for a given γ:

  $\iff$ B-brane transport.

- To get Φ^{γ}, we need to define B-branes on GLSM.

  (This is not as easy as one might imagine—)

Back to Ex. 1:

$$\begin{tikzcd}
 & MF_G(W) \arrow[dl, "\pi_2"'] \arrow[dr, "\pi_1"] & \\
MF_{\mathbb{Z}}(f_5) \arrow[ur, dashed, "\nexists \text{ nat. lift}"] \arrow[r, shift left] & \mathcal{T}_{\ell} \arrow[l, shift left] \arrow[r, shift left] \arrow[u, phantom, "\cup"] & D^b Coh(Q_5) \arrow[ul, dashed, "\nexists \text{ natural lift}"'] \arrow[l, shift left]
\end{tikzcd}$$

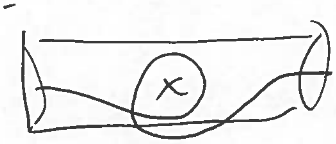
$\zeta < 0$ $\qquad\qquad$ $\zeta > 0$

So how to reconcile $\nexists$ nat. lifts w/ equivalences of LHS & RHS?

Some natural subset $\mathcal{T}_{\ell}$ indexed by ℓ. (kills redundancy)

the rule $\mathcal{T}_{\ell} \subset MF_G(W)$ is given by "Grade Restriction Rules."

"Window category" (& have infinitely many of them depending on path:



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