11/18/2016, Mauricio Romo, <u>Gamma class from GLSMs & B-brane transport II</u>

Recall,

$$\text{GLSM}: \quad \big(G,\ \rho_m : G \to SL(V),\ \underset{\substack{\text{weight 2} \\ \text{under } R}}{W},\ \{t_i\}_{i=1}^{s},\ R\big).$$

R ← R-charge (weights)

(if $rk(V) = N$, define $S := \mathbb{C}[\phi_1, \dots, \phi_N]$).

• B-branes on GLSM $\quad MF_G(W) \ni \mathcal{B}$; $\mathcal{B}$ is defined by

1) $M = M_0 \oplus M_1$ $\mathbb{Z}_2$-graded vector space (free S-modules)

compatible w/ ρ_m and R

2) $\rho_Q : G \longrightarrow SL(M)$ (<u>even</u> w.r.t. $\mathbb{Z}/2$ grading)

3) $r_* : \mathfrak{u}(1) \longrightarrow \mathfrak{gl}(M)$ (Diagonal, even (implied by diagonal))

$\mathfrak{u}(1) = \text{Lie}(U(1)_R)$

4) $Q \in \text{End}^{\text{odd}}(M)$ s.t. $Q^2 = W \cdot \mathbb{1}_M$.

<u>Compatibility</u> means

- $\rho_Q(g) \cdot Q(\rho_m(g) \cdot \phi)\, \rho_Q^{-1}(g) = Q(\phi) \quad \forall\, g \in G$.
- $\lambda^{-r_*} Q(\lambda^R \phi)\, \lambda^{r_*} = \lambda Q(\phi) \quad \forall\, \lambda \in U(1)_R$.

Denote $\mathcal{B} = (M, Q, \rho_Q, r_*)$ - $G \times U(1)_R$-equivariant M.F. of W.

A <u>physical B-brane</u> on the GLSM, requires specifying "boundary conditions for the fields."

⇝ also need to specify a contour $\gamma \subset \mathfrak{t}_{\mathbb{C}} = \text{Lie}(T_G)^{\mathbb{C}} \cong \mathbb{C}^{rk(G)}$, γ-Lagrangian,

s.t. there is an asymptotic condition on γ depending on a function

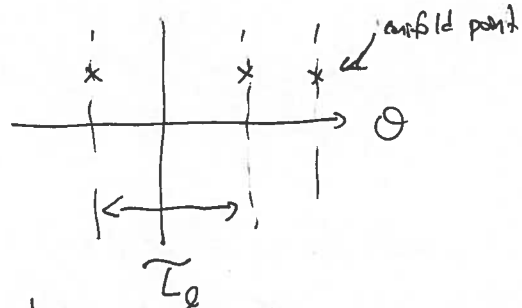
$$\text{Im}\big(\widetilde{W}_{\text{eff}}[\mathcal{B}]\big) : \mathfrak{t}_{\mathbb{C}} \longrightarrow \mathbb{R}.$$

$\widetilde{W}$ ~ a lift of W to $\mathfrak{t}_{\mathbb{C}}$ in a way depending on $\mathcal{B}$?

(Aside: recall last time:

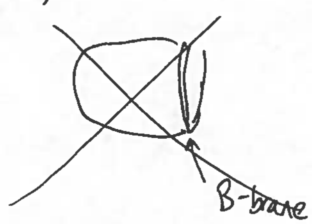
$$MF_G(W) \supset \mathcal{J}_e \xrightarrow[\sim]{\text{equiv. cat.}} D^b\text{Coh}(X),$$

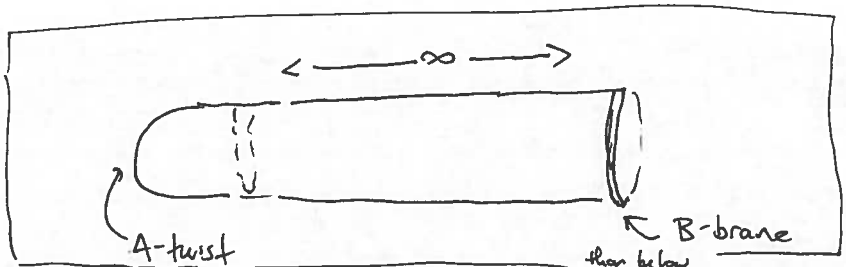
Today, will define central charge for a B-brane in GLSM, & is defined globally on all Kähler moduli)

(Look at covering of  : )

Given a pair $(\mathcal{B},\gamma)$, we can compute its <u>central charge</u> by putting the GLSM ~~GLSM~~ on D^2 ($\Leftarrow$ the worldsheet in this case) with boundary conditions depending on $(\mathcal{B},\gamma)$.

Schematically,



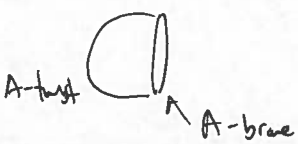


(*)

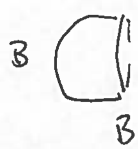
(Rmk: this physical coupling is "more natural" than below in the sense it preserves R-charge everywhere)

gives physically a central charge for $\mathcal{B}$. (not something one gets from the top. B-twisted theory).

Can also consider:



"disc invariants"; open GW ; not the same as above.



, but these are <u>different</u> from (*),

The formula is:

$$Z_{D^2}(\mathcal{B}) = \underbrace{\tilde{C}}_{\text{constant}} \int_{\gamma\subset \underset{\sim}{t}_{\mathbb{C}}} d^{l}\sigma \left(\prod_{\alpha>0}\alpha(\sigma)\sinh(\pi\alpha(\sigma))\right)$$

$$\times \prod_{q\in \text{weight}(\rho_M)} \Gamma\left(\overset{\sqrt{-1}}{i}\,q(\sigma)+R_q/2\right)e^{it(\sigma)}f_{B}(\sigma),$$

<u>and</u>

$$f_B(\sigma) = \mathrm{Tr}_M\left(e^{i\pi r_*}e^{2\pi\rho_Q(\sigma)}\right)$$

here, means weights of ρ_Q. (slight abuse of notation).

note: only depends on representations used to specify $\mathcal{B}$, <u>not</u> on the continuous parameter specifying $\mathcal{B}$.

where also:

- $\sigma \in \underset{\sim}{t}_{\mathbb{C}}$
- $l = \dim(T_G$
- α – positive roots of G.
- R_q – weights of R. ($0 < R_q < 2$ ← some physical restriction) $(**)$
- t as before
- $t(\sigma) := t^a \sigma_a$.

For this formula to make sense, γ must be an "admissible" contour: meaning

(i) Lagrangian in $\underset{\sim}{t}_{\mathbb{C}}$ (but of course, could wiggle γ a little so no longer Lagn & have same integral; so this could in principle be dropped: condition (i) comes from physical considerations).

(ii) Continuous deformation of

$$\gamma_{\mathbb{R}} = \{\mathrm{Im}(\sigma) = 0\} \subset \underset{\sim}{t}_{\mathbb{C}}$$

(i.e., doesn't hit the poles of the Γ-function)

$(**) \Rightarrow \gamma_{\mathbb{R}}$ doesn't hit the poles itself.

(iii) $Z_{D^2}(\mathcal{B};\gamma)$ should be absolutely convergent, e.g., if

function of t ← on kähler moduli (multivalued on kähler moduli, fcn. on $\tilde{\mathcal{M}}_K$).

$$Z_{D^2}(\mathcal{B}) = \int_{\gamma \subset \underset{\sim}{t}_{\mathbb{C}}} d^l\sigma\, F_{\mathcal{B}}(\sigma;t)\text{ , then } \Longleftrightarrow |F_{\mathcal{B}}(\sigma,t)| \to 0 \text{ in all asympt. directions of } \gamma.$$

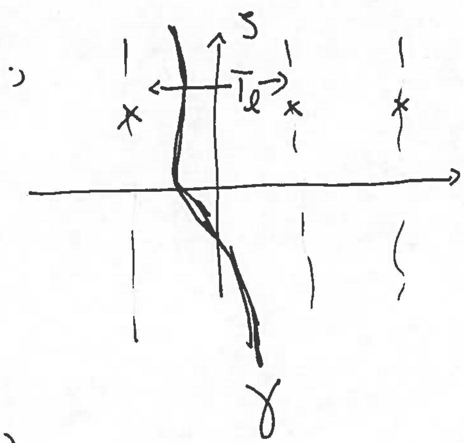
(iv) Weyl invariant.

(Conjectural relationship between above & grade restr./window categories)

Proposal: $\mathcal{B}$ is grade restricted (e.g., in window cat $\mathcal{T}_\theta$), iff $\exists$ an admissible γ_t for all $\zeta = \mathrm{Re}(t)$ and all $\theta \in T_l$

↑ some region inside $\mathrm{Arg}(\tilde{\mathcal{M}}_K)$ ← univ. cover. $\ni \theta$

e.g.,



((Q: is there an A-model / Fukaya-cat. theoretic interpretation of the γ's?)).

Aside: can write: using Euler formula (for Γ)

$$\left(\int_\gamma d^l\sigma \int_{\mathbb{C}} dt\, t^{-q(\sigma)+R_q/2-1}\, e^{\cdots}\right) = \int_{\mathbb{C}\times\gamma} e^{(\text{—})}$$

↑ Hori–Vafa potential.

Ex: if $G = U(2) = SU(2) \times U(1)$

then $t = \xi - i\theta$ (# t's = dim center of G)

& get $\int_{\gamma \subset \mathbb{C}^2} d^2\sigma \, e^{t(\sigma_1 + \sigma_2)} (\text{---})$

two integration vars. one kähler param. in this case.

Ex: case of a hypersurface: (deg. N hypersurface in $\mathbb{P}^{N-1}$

$$Z_{D^2}(\mathcal{B}) = \int_{\gamma \subset \mathbb{C}} d\sigma \left(\Gamma(-iN\sigma + 1)\, \Gamma(i\sigma)^N \right) e^{it\sigma} \times \left(\sum_{j=1}^{\dim M} (-1)^{r_j} e^{2\pi q_j \sigma} \right)$$

Can show $\exists$ γ admissible iff all weights q_j satisfy

$$-\frac{N}{2} < \frac{\theta}{2\pi} + q < \frac{N}{2}$$; and in this case,

$$\widetilde{W}_{\text{eff}}(\sigma) \sim (\xi - N \log N)\, \mathrm{Im}(\sigma) + \left[N\pi - \mathrm{sgn}(\mathrm{Re}(\sigma))(\theta + 2\pi q) \right] |\mathrm{Re}(\sigma)|.$$

(essentially linear).