I would like to point out a simple observation regarding the Gamma class $\hat{\Gamma}(X)$ of a compact complex manifold X.

Suppose we have

$$c(TX) = \prod_{i=1}^{\dim X} (1 + x_i).$$

Then we have the following formula for the logarithm of the Chern class

$$\log c(TX) = \sum_{i=1}^{\dim X} \log(1 + x_i) = \sum_{i=1}^{\dim X} \sum_{k \geq 1} \frac{(-1)^{k+1}}{k} x_i^k$$

$$= \sum_{k \geq 1} \frac{(-1)^{k+1}}{k} \sum_{i=1}^{\dim X} x_i^k$$

understood as taking (finite) logarithm of a unipotent element of an algebra.

On the other hand, for the Gamma class we have

$$\log \hat{\Gamma}(X) = \sum_{i=1}^{\dim X} \log \Gamma(1 + x_i) = \sum_{i=1}^{\dim X} \Big(-\gamma x_i + \sum_{k \geq 2} \frac{\zeta(k)(-1)^k}{k} x_i^k \Big)$$

$$= -\gamma c_1(X) - \sum_{k \geq 2} \frac{\zeta(k)(-1)^{k+1}}{k} \sum_{i=1}^{\dim X} x_i^k.$$

So the graded pieces of the log-derivatives of $c(X)$ and $\hat{\Gamma}$ differ from each other by multiplication by (the negative of) the Euler constant and the values of ζ.

Disclaimer: The Chern *character* of X also looks rather simple in this form; I also don't claim any particular geoemtric meaning behind these formulas.