Hori – Mirror Symmetry & Reality

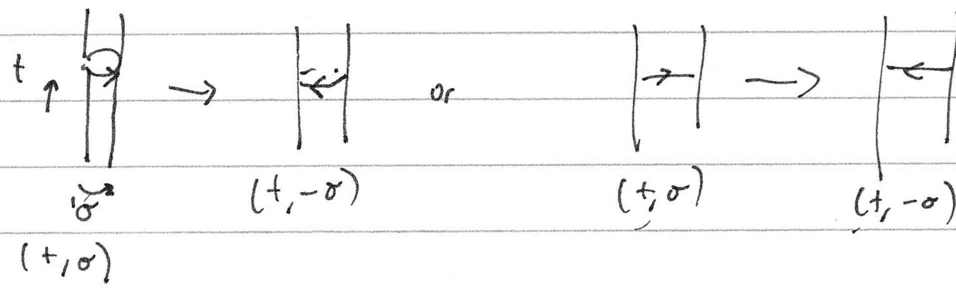
(M, τ) a space with an involution

$\longrightarrow$ "Orientifold"

by gauging $(x : \Sigma \to M) \longmapsto (\tau \circ X \circ \Omega : \Sigma \to M)$,

$\Omega : \Sigma \to \Sigma$ orientation reversing involution.

e.g. closed string theory: open string theory:



An orientifold plane $Y \subset M$ is a connected component of M^τ

N D-branes at Y

$$\begin{cases} O(N) \\ \\ USp(N) = Sp(N/2) \end{cases} \text{— the O-plane is of type } \begin{cases} O^- \\ O^+ \end{cases}.$$

Type II Orientifold:

M : 10 dim'l Riemann. mfold of signature $(9, 1)$ ← real & Weyl possible at same time.

with a spin structure.

τ : lifts to the real spinor bundle $\tilde{\tau} : S(M) \to S(M)$.

$\tau^2 = id$,

$$\tau : \text{ori.} \begin{cases} \text{preserving}, \ \tilde{\tau}^2 = \begin{cases} +id & (B_+) \\ -id & (B_-) \end{cases} \Big\} \ II\,B \\ \text{reversing}, \ = \begin{cases} +id & (A_+) \\ -id. & (A_-) \end{cases} \Big\} \ II\,A. \end{cases}$$

possible O-planes (codim $= k \to O(9-k)$-plane)

(B_+)	codim $= k = 0 \mod (4)$	O9/O5/O1
(B_-)	2	O7/O3
(A_+)	3	O6/O2
(A_-)	1.	O8/O4/O0.

D-branes in Type II string theory:

data: E : a hermitian v.b. on M.

Chan-Paton bdle.

A : a unitary connection of E

gauge field

T : a hermitian endomorphism of E.

tachyon

IIA: E ungraded

IIB: $E = E^0 \oplus E^1$, $\mathbb{Z}/2$ graded $A = \begin{pmatrix} A^0 & 0 \\ 0 & A^1 \end{pmatrix}$, $T = \begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$

Orientifold action of (E, A, T)

$A \mapsto -\tau^* A^t + \alpha$. $\quad E \mapsto \tau^* E^* \otimes \mathcal{L}$.

$T \mapsto \pm \tau^* T^t$

B-field

α is constrained by $d\alpha = B + \tau^* B$.

"worldsheet action" $\quad S_B = \int_\Sigma X^* B \longmapsto \int_\Sigma (\tau \circ X \circ \Omega)^* B = -\int_\Sigma X^* \tau^* B$

e^{iS_B}

$$\Delta S_B = -\int_\Sigma X^* (B + T^* B).$$

If $\partial\Sigma = \phi \Rightarrow$ need $e^{i\Delta S_B} = 1 \iff [\tau^* B + B] \in H^2(X, 2\pi \mathbb{Z})$

i.e. $[\tau^* B + B] = -2\pi c_1(\mathcal{L})$ for some cplx. l.b. $\mathcal{L}$ on M

If $\partial\Sigma \neq \phi$, $\Delta S_B = -\int_\Sigma X^* d\alpha = -\int_{\partial\Sigma} X^*\alpha$.

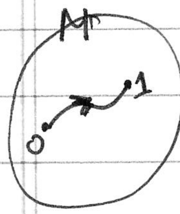
This leads to the shift of A

Eq. (:) A enters into e^{iS} as the parallel transp. $P\exp\left(-\int_{\partial\Sigma} X^* A\right)$

Orientifold action on open string states

An open string state Φ is the assignment $X \in Map([0,1], M) \longmapsto \Phi[X] \in$ ~~Hom~~

$\in Hom_{\mathbb{C}}(E_{X(0)}, E_{X(1)})$



(actually some other stuff, hidden).

A natural action of parity

$$\Phi(X) \mapsto P(\Phi)[X] = \Phi[\tau \cdot X \circ \Omega]^t$$

Mirror symmetry:

$$M = X^{2n} \times \mathbb{R}^{(9-2n)+1}$$

Kähler, σ-model on X has $\mathcal{N}=(2,2)$ supersymmetry

$\tau : X \to X$ preserves a half of SS

(B) τ: holomorphic

(A) τ: anti-symplectic

category w/ duality.

Mirror symmetry:

$$(X, \tau, B, \mathcal{L}, \alpha, c) \longleftrightarrow (X', \tau', B', \mathcal{L}', \alpha', c')$$

holo. antisympl.