Discussion:

- $\mathcal{M}$ alg. ∞-stack.
- $\exists$ stratification of $\mathcal{M}$ whose open strata are schemes

means:

~~$\mathcal{M}$~~ $\mathcal{M} \supset \mathcal{M}_\alpha \supset \mathcal{M}_{\alpha-1}$, where

for alg 1-stacks

$$\begin{tikzcd} \mathcal{M}_{\alpha-1} \arrow[r, hook] & \mathcal{M}_\alpha \\ S' \arrow[u, "L"] \arrow[r, hook, "\text{closed}"'] & S \arrow[u] \end{tikzcd}$$

is closed & ~~$\forall$~~ $(\mathcal{M}_\alpha - \mathcal{M}_{\alpha-1}) \to \exists$ scheme X_α.

S = scheme

pull-back.

locally gerbe over like BGs.

universal for maps towards schemes

(fibers are higher automorphisms, have same points)

$\mathcal{B}$ dg algebra of fin. type $/k$

$\mathcal{M}_\mathcal{B}$ = all $\mathcal{B}$-dgmod perfect $/k$

2) $\mathbb{Z} \longrightarrow \mathbb{N}$ with finite support

$$\mathcal{M}_\mathcal{B}^{\nu} = \{ E \in \mathcal{M}_\mathcal{B} \mid \forall k \to k(p) \ \dim_{k(p)} H^i(E \otimes_k k(p)) \leq \nu(i) \}$$

of finite type

$H^i(E \otimes_k k(p)) = \mathrm{Ext}^i_X(E_0, E)$ — E_0 compact gen. of $D(X)$

Rel'n between saturatedness for dg vs. triangulated categories?

- k a field
- $\mathcal{B}$ dg alg. $/k$.

$\mathcal{B}$ sm. proper $\Rightarrow D_{pof}(\mathcal{B})$ saturated in the sense of Bondal-Van den Bergh.

$\Leftarrow$? Open question

Known: If $D_{pef}(\mathcal{B})$ and $D_{pef}(\mathcal{B} \otimes^\circ \mathcal{B})$ are saturated in the sense of Bondal-vd.-B, then $\mathcal{B}$ is smooth & proper.

(Ref: (–, Vaquié, "Algebraization of –)

finite cell dg algebra B

$$\begin{tikzcd} k = B_0 \arrow[r] & \cdots \arrow[r] & B_i \arrow[r] & B_{i+1} \arrow[r] & \cdots \arrow[r] & B \\ & & k\langle x\rangle \arrow[u] \arrow[r] & k\langle x,y\rangle \arrow[u] \arrow[ul, phantom, "\ulcorner"] & & \end{tikzcd}$$

$\deg y = \deg x - 1$
$d(y) = x.$

Prop: B is of finite type $\Leftrightarrow$ it is a retract in $\underline{Ho}(\underline{dg\ alg}\,|\,k)$ of finite cell algebra.
can do explicitly..

Consequence: B of finite type$/k$ (saturated) $\Rightarrow$ $\exists\ k'$ finite type$/\mathbb{Z}$, $k' \to k$, B' of finite type $/k'$ (saturated) s.t. $B' \otimes_{k'} k \simeq$.

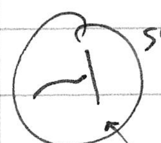
Kontsevich: A saturated
$M \in Perf(A)$
kill M.

finite diagram of finite algs.
then holim again of finite type.

$$\begin{tikzcd} pt \arrow[r, "\text{model of}"'] \arrow[d] & A\text{-mod} \\ 0 & \end{tikzcd}$$
$\overrightarrow{A\text{-modules}}$

Q: can one get all fin-type cats. in this way?
like a res. of singularities.

S separated of fin. type, then
$D^b(Coh\ S) = Perf(A\text{-mod})$?
$\cup$
$Perf(S)$.


smooth proper variety

complement of ∂ set, get M, fin. type do all fin. types arise in this way?

localize finite type dga is still fin. type
(commutativity of fin. lims w/ filtered colims).

$\exists$ Noetherian type (a.c.c.) conj. for saturated categories.

A saturated.

M obj. $\in$ Perf(A)

~~fexat~~

A - dga

M - dg mod.

$$B = \underline{R\mathrm{End}}_{A\text{-mod}}(A)$$

$$\hat{A} = R\mathrm{End}_{B^{op}\text{-mod}}(M)$$

repeat twice, get again B.

ex. $A = \mathbb{C}[x]$

$M = \mathbb{C}$. , same for n-variables

$B = H^\bullet(S^1)$

$\hat{A} = \mathbb{C}[[x]]$

(for A affine scheme, M coh sheaf).

Conj: on a smooth scheme (or other type), do double centralizers, get functions on formal completion of support.

in n.c. sense