Key: $Fs = A$, B = full image under r.

$$Fs \xrightarrow{r} B.$$

$r^*: B \to A$-mod.

$r^* B := A \otimes B / A$.

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non-comm.

Barannikov – Developments in the BV formalism

$S_{0,1}$ – A_∞-alg. w/ (even/odd) scalar product, so S – multiloop higher genus generalization of A_∞-alg.

$$\hbar \Delta S + \frac{1}{2}\{S, S\} = 0$$

$$\Leftrightarrow \Delta(\exp \tfrac{1}{\hbar} S) = 0$$

not D-M compactification, Kontsevich compactification.

solutions to nc BV-eqn have characteristic homology classes in $H_*(\bar{M}_{g,n})$,

answers question of M. K. : find alg. str. extending Massey of A_∞-alg. in $H_*(M_{g,n})$ to $H_*(\bar{M}_{g,n})$. + Witten – combin. model for compactification of $M_{g,n}$.

---

high genus analogue for theory of variation of (nc) Hodge str (of CY-type)

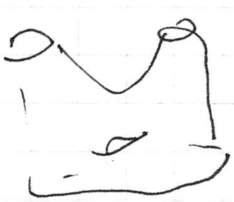
A_∞-periods, nc-VHS $(HC_* \subset HP) \to (H_*(\bar{M}_{0,n})$-action$)$ on HH

- A_∞: Σ trees $\longleftrightarrow$ nc-BV: Σ graph

Feynman transform of "modular operad" – Kapranov–Getzler.

~~[illegible]~~ dim $M/2$ even: $\Lambda(\mathbb{C}^n)$ $\left[\text{sol'ns to: } \Delta(\exp \frac{S}{\hbar}) = 0 \right]$ ← BV eqn.

dim $M/2$ odd: $S(\mathbb{C}^n)$



$S_{g,i}$ = # (genus g holom. curves w/ i bd. components)

$(\Sigma, \partial \Sigma, p_i) \to (M, L_i, \oplus L_i \cap L_j)$

vector space is $\oplus H_*(L_i \cap L_j)$

Not proved

$\to H_*(\bar{\mathcal{M}}_{g,n})$

· stable ribbon graph ~~[illegible]~~

(k, with compute).

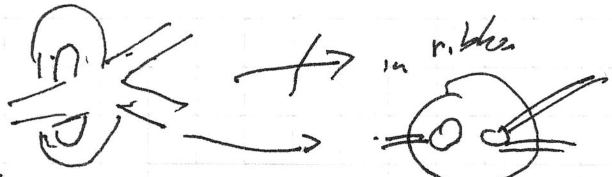
$S = \sum t^{g} S_{g,i}$

· vertex v → $\sigma \in \mathrm{Aut}(\mathrm{Flags}(v)) \to S_{g,i} \in$ → Sym → A

S_i · edges → β.

$W_\Gamma: \prod_v S_{g,i} \otimes \prod_{\text{edges}} \beta^{-1} \longrightarrow k.$

claim: nc BV-eqn. $\iff \left(\sum_{\text{stable } \Gamma} W_\Gamma[\Gamma] = 0\right)$ in stable graph cplx



HH_* HH^*

$A \to B \to C \Rightarrow A$

ow $\Rightarrow A = B$.

$HH^* \times HH_* \to HH_*$?

or HH^*?

M.K. 9/2 Chern Simons ⤳ via commutative graph cplx.

{ no edge on flags. }

similar story ~~of ribbon graphs~~ for assoc. algebras — working w/ ribbon graphs

Ass algebra w/ scalar product $\to H_*(M_{g,n})$

~~assoc. alg. w/~~ dg assoc. alg. w/ $H_*(d) = 0$ odd scalar product $\longrightarrow H^*(M_{g,n})$

$\hat{W}_\Gamma$, then

$d(\sum \hat{W}_\sigma^{\text{dual}} \sigma) = 0$

$(= \sum_{\text{contributions from vertices w/ loops}})$

$\sigma_i \longrightarrow \mathrm{Tr}(a_1 \ldots a_n)$

edges → homotopy operator s.t. $[h, d] = \mathrm{id}$.

(before, had $(m_n(\ldots), a)$)

~~[illegible]~~ X smooth, C-Y.

is: $HH_*(W(M^4))$ combinatorial?

$R\mathrm{Hom}(G[i], \cdot)$ maps f_i to 0

$X_1 \xrightarrow{f_1} X_2 \xrightarrow{f_2} \ldots X_n$

$R\mathrm{Hom}(G[i], \cdot)(f_i) = 0$

$HH^* \to HH_*$

$HH_* \longrightarrow HH^*$

$\cup$ sH

$(p(\alpha) \cup p(\beta))$ mod