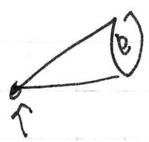
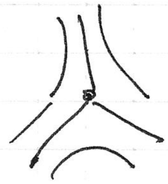
Discussion [Kontsevich]:



what goes here?



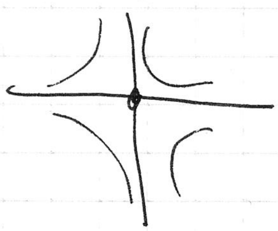
how to get A_2?

other examples?

relation of spine w/ vector field.

---

ex. L_1

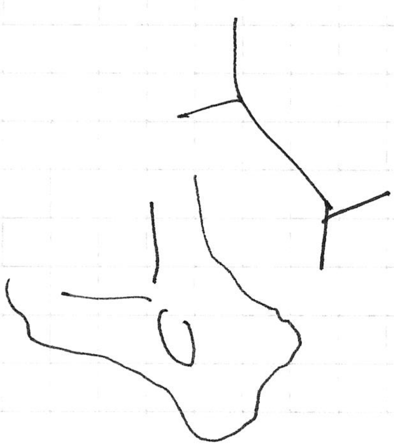


L_2

$(i\mathbb{R})^n$

$\mathbb{R}^n$.

make small nbhd, define a little



$(\mathbb{R}^2 \setminus \text{Disc}) \cup (i\mathbb{R}^n - \text{disk}) \cup \text{disk}$

Calabi-Yau structures

A dg algebra unital

Higher Hochschild cohomology

$$HH^{(k)}(A) \quad k = 1, 2, \ldots$$

$$C^{(k)}(A)$$

$$HH^{(k)} = R\mathrm{Hom}_{A^{\otimes k}\text{-mod-}A^{\otimes k}}(A^{\otimes k}, \underset{\text{cyclic permutation}}{A^{\otimes k}}) \quad \mathbb{Z}/k$$

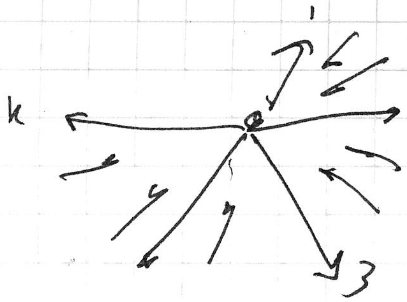
Hom from (id, permutations)

Manifestly Morita-inv. definition

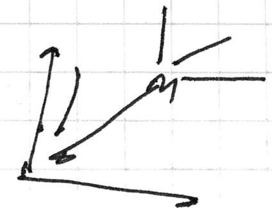
$k = 1$: usual HH^*.

$$C^{(k)}(A) = \prod_{n_1, \ldots, n_k \geq 0} \mathrm{Hom}(A[1]^{\otimes(n_1 + \cdots + n_k)}, A^{\otimes k})$$

any bimodule



Differential



Same formula for homology is same for any k. Different for cohomology!

Choose even or odd CY structure. Twist permutation by $(-1)^{(k-1)\cdot \dim CY}$.

action of cyclic group:

$$\prod_{k\geq 1} C^{(k)}(A,A)^{\mathbb{Z}/k\mathbb{Z}}$$ will be a Lie algebra; commutator of (k,l) is $k+l-1$.

"necklace Lie algebra"

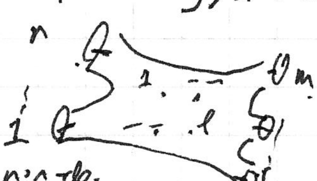
for $C^{(1)}$ solutions of $[a,a]=0$ is A_∞ algebra, + other stuff which could be 0.

$\tilde{m} :=$ Solutions of MC in $\left(\prod C^{(k)}\right)^{\mathbb{Z}/k}$ gives A_∞ str. on A + a pre-Calabi-Yau structure (which could be 0). Think of as a section of anti-canon. bdl.

theorem: (?): For A w/ pre CY, get

$$HH(A)^{\otimes n} \otimes H_\bullet(M_{g,\vec{n}+\vec{m},l}) \longrightarrow HH(A)^{\otimes m}$$

homology; marked pts. on inside. $n, m \geq 1$



w/ no finiteness, nondeg. assumptions whatsoever!

(for smooth, proper, can extend n or m down to 0 respectively)

Graphs

ribbon oriented + acyclic*, connected, $\neq \emptyset$. • m outputs . $\deg_{out} = 0$.

vertices = inner vertices

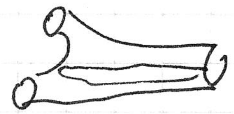
• $\deg_{out} \geq 1$.

if $\deg_{out} = 1$, then $\deg_{in} \geq 2$. numbered $\approx \{1, \dots, m\}$

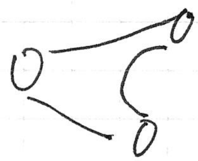
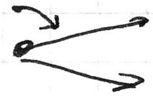
2-dim cells $\approx \{1, \dots, n\}$.

+ stuff for tangent directions

Differential: <u>add extra edge</u>.

$g = 0$:  , then graph is .

(for marked pts on bd, need more)

surface like this:  $\rightsquigarrow$ 

this version has no marked pts. on ∂.

so acts on cyclic.

acyclic, ~~no~~ trace, etc.

<u>non-trivial fact</u>: this is really controlled by cohomology <u>of moduli space</u> (open part).

not sure how to do this

Suggests cell decomp. of Σ-pt. compactification of $M_{g,n+m} \times \Delta^{n-1}_{int}$.

- what about non-degeneracy conditions?

Non-degen:

$$\tilde{m} = \tilde{m}^{(1)} + \tilde{m}^{(2)} + \tilde{m}^{(3)} + \dots$$

$$\tilde{m}^{(k)} \in C^{(k)}$$

$m^{(1)}$ for product, $m^{(2)}$ for non-degeneracy. (for general k, $(A^*)^{\otimes k-1}$)

2 cases:

A is proper, then $HH^{(2)} = RHom_{A\text{-}A}(A^*, A)$ (A^*: linear dual.)

A is smooth, $HH^{(2)} = RHom_{A\text{-}A}(A, A^!)$

CY is ~~these~~ ~~given~~ $\tilde{m}^{(2)}$ are quasi-isos.

(yesterday was HH_*, i.e. $RHom(A^!, A)$).

if you get either type of CY, then

S' is ~~a~~ (graded) trivial, depending on parity of dimension.

If smooth & proper, both these non-deg's are equivalent.

For smooth, proper, & degen. of HdR (choose a splitting), can replace $m, n \geq 0$ and replace $\mathcal{M}_{g,n+m}$ by $\overline{\mathcal{M}}_{g,n+m}$.

So point here, is that in complete generality have field theory, just non-unital.

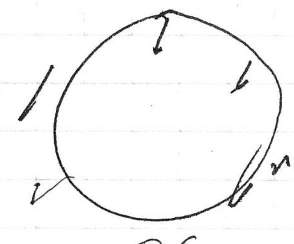
---

have $HC^- \longrightarrow HH$ essentially, choose a splitting, want to be an epi, essentially degeneracy.

(e.g. Fano w/ section of anticanon. bundle, get B-model, interesting to see what A-model will be).

---

what's structure on ~~these prop~~ $HH^{(k)}$? big PROP acting on all of this? — generalization of Deligne conjecture.



Hopf-Lurie, surface w/ involutions, i.e. non-0 v.f.

$C = \mathrm{Curve}/\mathbb{C}$. $s \in \Gamma(K^{\otimes 2})$, get $\mathbb{Z}$-graded cat $F(C)$, get stability conditions on $F(C)$

related to 3-valent graphs?

$C^{(k)}$ depends only on cont. pts? are these boundary str. for each acting A?

acyclicity of vector field.

Serre functor not generally invertible! for cpct get S, for smooth, get S^{-1}.

For saturated, can do inverse; but $HH^{(-k)} = (HH^{(k)})^*$.

$\boxed{Q!}$ Is there a notion of non-deg. w/o smooth or proper, interpolating between both? A: Don't know...

?Q!: Does this live alg. control. deformations of anything? Don't know.

Q! From talk today, had F, $F^2 = 1$, $F^+ = F$. Is this the only such one? Is there an alg. w/ non-trivial cyclic homology??

$\mathcal{H}, F$

$$A_N = \{ a \in \mathrm{End}\,\mathcal{H},\ \text{s.t. } \mathrm{Tr}([a, F])^N < \infty \}$$ an algebra.

Consider A_N as alg. over $\mathbb{Q}$, take HP. It's huge! get some numbers... if alg. over $\mathbb{C}$, what's CC? not known. or HH_*.

General thing, relates to talk 1:

Singular Lag'n in

U Liouville mfld $\longrightarrow$ wrapped Fukaya category $/\mathbb{Z}$

not speak about Symp grp, but instead Universal family.

π_1 (Univ. family) $\longrightarrow \mathrm{Aut}(\mathcal{W})$.

big, e.g. for >–<, get Teichmüller grp.

More: can multiply U by infinite interval, corresponds to L.G. model
doesn't change $\mathcal{W}$ $\quad \mathbb{C}, z$?.

Consider stable symplecto group, could be in principle computable from topology,
as we multiply.

e.g. $\mathrm{diff}(D^2) \to \mathrm{diff}(D^4) \to \cdots \mathrm{diff}(D^\infty)$.

Waldhausen. these are hard

in ∞, get K theory of integers?? $\langle K_0 \; --- \rangle$

<u>Maybe something</u> can be done here?

<u>Q</u>: Is there a chance to be stable BO?

i.e. from category, construct inf dim'l sympl. manifold

e.g. Triangulated cat / local field.

$\rightsquigarrow$?

∞-dim. symplectic manifold??

Ref: Berkovich spectrum (space of pts) of triangulated category,
projective limit of finite polytopes, finite htpy type,
very heavy.

How to get symplectic manifold??

abelian variety: cat./local field $\to$ $\mathrm{tors}_{\mathbb{R}} + \mathrm{tors}_{\mathbb{R}}^{\vee}$, natural symplectic structure from ((pseudo? of commuting endofunctors??))

tangent space is HH^*, Gerstenhaber bracket

pieces of non-commuting space, commuting symmetry, get pieces of symplectic manifold.

So many examples, should be something behind it.