Conj: ① $\{b \mid \nabla\Phi(b) = 0\} = \mathcal{M}(L)$ Maurer-Cartan scheme

② $\Psi_J = \Phi$ on $\mathcal{M}(L)$

J is S^1 invt. cplx. str.

~~To make sense~~. Need to compare Lau's & Fooo's perturbations to prove this.

Keller – The Periodicity Conjecture via CY Categories:

mathematical physics

periodicity conjecture: applications to computations of central charge in CFTs & dilogarithm identities

Al. Zamolodchikov, 1991

proof based on homological algebra, study of certain 2-CY triangulated cats

philosophy of proof: $\boxed{\text{categorification}}$

ingredient 2: theory of cluster algebras (Fomin–Zelevinsky, 2002), interface between combinatorics & categorical setup

Plan: 1. The conjecture

2. The beginning of the proof: the categorifications of finite reduced root systems

3. The end of the proof: homological periodicity.

4. Dessert: quiver-version of the conjecture.

1. The conjecture

Δ, Δ' Dynkin diagrams (simply laced) i.e. type A, D, E

vertices: $I = \{1, \dots, n\}$, $I' = \{1, \dots, n'\}$

Cox. numbers h resp. h'. order of coxeter elt, see table!

Δ	h
A_n	$n+1$
D_n	$2n+2$
E_6	12
E_7	18
E_8	32

Incidence matrices: A resp. A' so $a_{ij} = \begin{cases} 1 & \exists\, i - j \\ 0 & \text{else.} \end{cases}$

Associated Y-system

variables: $Y_{i,i',t}$, $i \in I$, $i' \in I'$, $t \in \mathbb{Z}$.

equations:

$$Y_{i,i',t-1} \cdot Y_{i,i',t+1} = \frac{\prod_{j=1}^{n} (1 + Y_{j,i',t})^{a_{ij}}}{\prod_{j'=1}^{n'} (1 + Y_{i,j',t}^{-1})^{a'_{i'j'}}}$$

(i,j')

$(i,i') - (j,i')$

Conj: All solutions are periodic of period dividing $2 \cdot (h + h')$.

[Gabriel – Happel: categorification of root systems]