Mikhalkin: (p,q) homology classes in tropical geometry

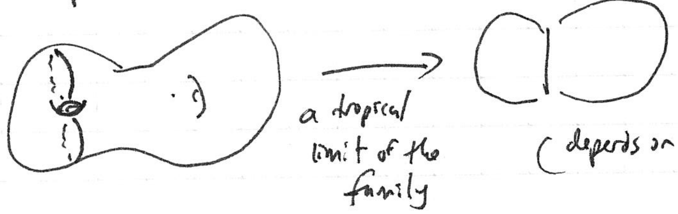
joint w/ I. Itenberg & I. Zharkov. | Katzarkov

(goal: translate mixed hodge str. of schemes to trop. geom.).

X - tropical manifold $\longrightarrow$ Homology/cohomology with bigrading.

so that dim (p,q)-groups $= h^{p,q}$ (generic fiber of a cplx. 1-param. family approximating X).

E.g.: Suppose we have a family $\mathcal{X}_t$ $t \in \Delta^*$.



(depends on parametrization t).

Most well-studied case: toric geometry:

$\mathbb{C}\mathbb{P}^n$ ← moment map

$\downarrow \mathrm{Log}_t$ for $t \gg 0$

$\mathbb{T}\mathbb{P}^n$ = simplex

$\mathbb{T}\mathbb{P}^n$ glued from $\mathbb{T}^n$, $\mathbb{T} = [-\infty, +\infty)$

"$x+y$" $= \max\{x, y\}$

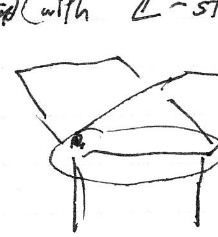
"xy" $= x+y$

Trop. Laurent polynomials w.r.t. "+," "·" define ◯ (Euclidean topology)

Can be geometrically rephrased as an $\mathbb{Z}$ integer affine structure on trop-dimen. strata.

(interested in trop. mflds smooth in the coarse sense.).

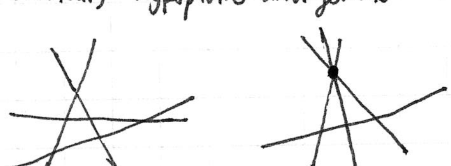
X^n - union of convex n-polyhedra ~~equipped~~ (with $\mathbb{Z}$-structure)
polyhedral complex.

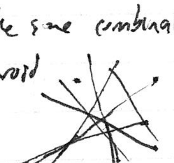
e.g. $n=1$  $n=2$ 

X smooth trop. manifold in coarse sense if at each point, it is given by a matroid-
⇝ cones from a simply balanced emb. to $\mathbb{T}^N$.

matroids = (virtual) hyperplane arrangements in $\mathbb{P}^n$

(also, can make some combinatorial rules that are not realizable)



ex. Fano matroid 

e.g.  non-realizable over $\mathbb{C}$.

$\Rightarrow$ all matroids give possible local models for junction pts. ~~in~~ a trop. mfold.
(Bergman, Ardila - Klivers, —)