Schapira II

Two perspectives on $\dot{T}^*M$:

$\dot{T}^*M$ — symplectic ω_M

$\overset{\shortparallel}{}$

$T^*M \setminus T^*_M M$ — homogeneous sympl. (α_M profile)

conic for $\mathbb{R}^+$-action

Converting non-conic to conic:

$$T^*M \times \dot{T}^*_{\tau>0}\mathbb{R} \longrightarrow T^*M$$

$$(t,\sigma) \in T^*\mathbb{R}. \quad (x,\xi,t,\sigma) \longmapsto (x, \xi/\sigma)$$

Microlocal th. of sheaves $\dot{T}^*M$ homog. sympl. mfold

Complex case X compl. mfold

$\dot{T}^*X$ — conic case: microdiff'l operators — Kashiwara '96

— non-conic case $/\hbar$ — Kontsevich (non-homog., Poisson), 2000.

DQ-modules

(use stacks, at least sheaves of categories?)

Our case: M real (C^∞, real analytic?)

T^*M

Nothing about how to glue (don't know yet).

(Kashiwara – S)

81 – 85 – 90

Astérisque — sheaves on m'folds.

k nice ring $\mathbb{Z}$ or a field.

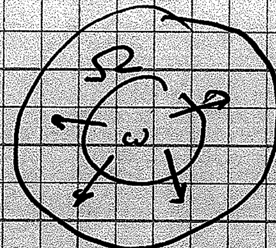
Work in $D^b(k_M)$. Assume comfort w/ six operations, etc.

sheaves of k-modules over M

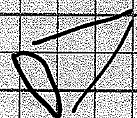
A few words on propogation:

L a local system on $\mathbb{R}^n$, two convex sets:

By propogation, $R\Gamma(\Omega; L) \simeq R\Gamma(\omega, L)$.



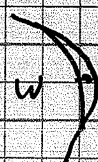
Another ex: $\Box u = 0$ $\quad \mathbb{R}^n$ (wave eq'n)



propogation in non-hyperbolic direction.

ex: $0 \longrightarrow \mathbb{C}_X \xrightarrow{P} \mathcal{O}_X \longrightarrow 0$ (diff'l operator)

$f \in \mathcal{O}(w)$, $Pf = 0$. Then have propogation in non-characteristic direction.



Singular support: $\quad F \in D^b(k_M)$. $\quad SS(F) \subset T^*M$.

Def: $p \notin SS(F)$ if $\exists$ $\mathcal{U} \ni p$ $\mathcal{U}$ open in T^*M

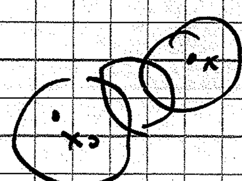
$\forall\ \varphi : M \longrightarrow \mathbb{R}$ $\quad \forall\ x_0 \in M$. s.t. $(x_0; d\varphi(x_0)) \in \mathcal{U}$.,

$$\left(R\Gamma_{\{\varphi \geq \varphi(x_0)\}}(F)\right)_{x_0} = 0 .$$

Idea:

φ defines a half space $\varphi \geq \varphi_0$.

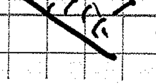
$x_0 \notin \operatorname{supp} F$?



means $\quad R\mathcal{H}om(k_{B(x,\varepsilon)}, F) = 0 \quad \forall\ |x - x_0| \ll 1$

$0 < \varepsilon \ll 1$.

If we're interested in cotangent bundle, don't take balls, we'll take lens



, test sheaf on all sufficiently near lens.

Thm: $\overset{\supset U}{M}$ open in $\mathbb{R}^n$.

γ closed convex proper cone in E.

$F \in D^b(k_M)$

polar cone?

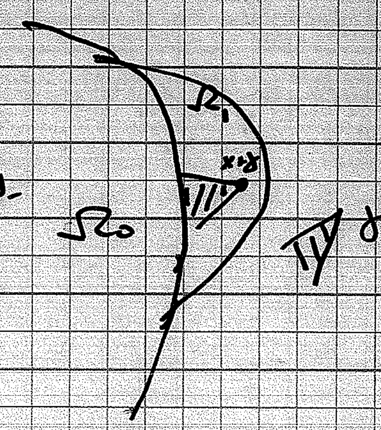
Assume $SS(F) \cap (U \times \mathrm{Int}\,\gamma^{\circ a}) = \emptyset$

in $E \times E^*$

$\Omega_0 \subset \Omega_1$

$\Omega_1 \setminus \Omega_0 \subset U$.

$\forall x \in \Omega_1$, $(x+\gamma) \setminus \Omega_0 \subset\subset \Omega_1$ e.g.



then: $R\Gamma(\Omega_1; F) \xrightarrow{\sim} R\Gamma(\Omega_0; F)$

$SS(F) \subset T^*M$.

- closed
- $\mathbb{R}^+$-conic
- "additive" $F_1 \to F_2 \to F_3 \xrightarrow{+1}$

$\Rightarrow SS(F_i) \subset SS(F_j) \cup SS(F_k)$, $j \neq k$.

- $SS(F)$ is co-isotropic

① M connected, F local system $F \neq 0$,

then $SS(F) = T^*_M M$.

② N closed submanifold;

then $SS(k_N) = T^*_N M$ conormal bundle.

Rmk: - gives a def'n of conormal for any loc. closed subset.

A locally closed

$T^*_A M := SS(k_A)$