Kontsevich III <cont'd>

Ex 2: M^3, $\partial M^3 = \Sigma^2$

$\mathrm{Rep}(\pi_1(\Sigma^2), G)$ ⊃ $\mathrm{Rep}(\pi_1(M^3), G)$

$\cong (\mathbb{C}^*)^{2n}$ — K_2 Lagrangian.

(should be directly provable, though known e.g. in physics. related to Aganagic's talk).

$$f(t) = \frac{\sqrt{1-4t} - 1}{2t} = 1 - t + 2t^2 + 5t^3 + \cdots$$

$\pm$ Catalan #'s

$f = 1 - tf^2$, $[f] \wedge [t] = 0 \in K_2(\text{alg. curve})$

write $f = \prod_{n \geq 1} (1-t^n)^{c(n)}$, $c(n) \in \mathbb{Z}$. Claim: $\Omega(n) = \frac{c(n)}{n} \in \mathbb{Z}$!

In fact, it is some DT [quiver with loop] w/ zero potential.

$$\Omega(n) = \# \left(\{ A \subset \mathbb{Z}/2n\mathbb{Z} \mid \# A = n, \sum_{a \in A} a = 1 \bmod n \} \middle/ \mathbb{Z}/2n\mathbb{Z} \text{ rotations} \right)$$

(1st proof) $[f] \wedge [t] = \prod (1-t^n)^{c(n)} \wedge [t]$

$$= \sum_{n \geq 1} \frac{c(n)}{n} [1-t^n] \wedge [t^n]$$

Playing w/ Steinberg relation, it's an element of (make all invertible)

$K_2(\mathbb{Z}[[t]]/t^M)$ computed $K_2 = \prod_{n=1}^{M} \mathbb{Z}/n\mathbb{Z}$.

this element is 0 in K_2, but also equals $\sum c(n) \cdot$ generators, they don't talk to each other.

(Proof 2): $K_2(F) = \mathrm{Ext}^2(\mathbb{Q}(0), \mathbb{Q}(1))$.

$K_2(F) = \mathrm{Ext}^2(\mathbb{Q}(0), \mathbb{Q}(1))$

$L \subset (\mathbb{C}^*)^{2n}$ K_2-Lagr. $/\mathbb{Q}$.

$p_1 \cdots p_n \; q_1 \cdots q_n$

$\sum [p_i] \wedge [q_i] |_L = 0$. On L, consider extension

$\mathbb{Q}(0) \equiv \oplus_i \mathbb{Q}(1) \equiv \mathbb{Q}(2)$ } triple extension

p_i $\qquad$ i $\qquad$ q_i

Get a motivic VHS $\Rightarrow$ mod p, has Frobenius, satisfies divisibility property, and criterion for divisibility $\Rightarrow$ proof.

mod p: $\log p_i = \frac{\partial F_0}{\partial \log \xi_i}$. $\quad F_0 = \sum \Omega(\gamma) \log(\xi^\gamma)$ (polylogarithms)

~~Can write f = 1 - t(f)~~

Can write $f = 1 - t(f)^k$ for any k ⇒ get some divisibility property.

∀ quiver Q ⟶ canonical φ_Q.

Suppose have $\varphi: \mathbb{C}[[x_1, \dots, x_N]]$ ↻ preserving same Poisson $\{x_i, x_j\} = a_{ij} x_i x_j$

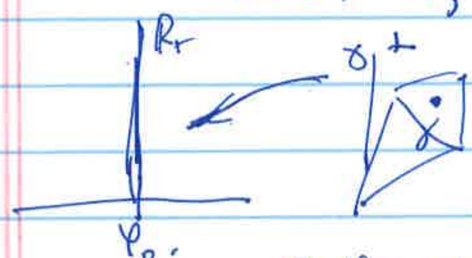
Then DT invts depend on stability.

But want to define $\Omega_\pm(\gamma)$ in following way:

$\gamma \in \mathbb{Z}^N_{\geq 0} \to \Omega$

Special stability condition s.t.

$Re(z) = \langle \gamma, \cdot \rangle \quad Im(z) > 0$



very degenerate stability.

In this subgroup, multiples of γ form a center.

Get coeffs in canonical picture.

(two choices, based on orienting hyperplane → give +/−)

So, gives $G \longleftrightarrow$ (collection of numbers $\Omega_+(\gamma)$)

$\longleftrightarrow$ (collection of $\Omega_-(\gamma)$.)

Suppose $\Omega_+(\gamma)$ has finite support, & #'s = 1.

Q: Is it algebraic or not?

Is this transformation is ~~not~~

the same as the quiver w/ generic potential transformation