Kontsevich II

K non-arch. field,

$$|\cdot| : K \to \mathbb{R}_{\geq 0}$$

$$|a \cdot b| = |a|\,|b|$$

$$|a+b| \leq \max(|a|, |b|).$$

non-trivial: Image of $|\cdot| \not\subset \{0,1\}$

complete: $\sum_{1}^{\infty} x_i \;\exists$ if $|x_i| \to 0$.

e.g. $\underline{k}((t))$, w/ $\left|\sum_{i \geq i_0} a_i t^i\right| = \frac{1}{2^{i_0}}$ $a_{i_0} \neq 0$.

$\mathbb{Q}_p$, Novikov field.

Analytic spaces:

e.g. polydisc: $r_1, \dots, r_n > 0$.

$\mathcal{O}(B(0,r_1) \times \dots \times B(0,r_n))$ Gives a Frechet algebra

$$= \Big\{ \sum_{i_j \geq 0} c_{i_1 \dots i_n} x^{i_1} \cdots x^{i_n} \;\Big|\; c_{i_1 \dots i_n} \in k,$$

$$\forall\, 0 < r_1' < r_1, \dots, 0 < r_n' < r_n,$$

$$\sup_{\text{indices}} |c_{i_1 \dots i_n}| (r_1')^{i_1} \cdots (r_n')^{i_n} < \infty$$

$\mathcal{O}(\ldots)/\text{closed ideal} = \mathcal{A}$. : typical local model of non-Arch space.

Spec $\mathcal{A}$ = (1) {multiplicative semi-norms $|\cdot|_x$ on $\mathcal{A}$ w/ same identities, s.t. $|\lambda|_x = |\lambda|\;\forall \lambda \in K$ constants

(2) $\{\mathcal{A} \to K'$ non-arch field $K' \supset K$
K linear, w/ all $\|_{K'}|_K = \|\cdot\|$, & K' minimal (image dense).

Spec A

Hausdorff locally compact space,

& have a sheaf of algebras (Berkovich) $K = \underline{k}((t))$

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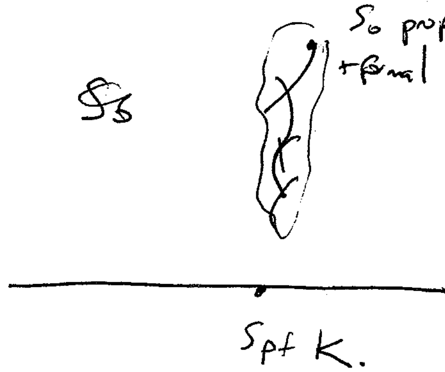
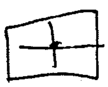
S – proper scheme $/K$.

S_0 proper scheme over $\underline{k}$ – formal thickening of scheme S at f_0

$\downarrow$

Spec f $k[[t]]$





$S^{an} \xrightarrow{\text{proper}} \Delta$ Divisors Clemens polytope ?? "Reynaud theory" approximate everything by simplicial complexes.

$\forall\ x \in S^{an}(\bar{K})$

$m_i \in \mathbb{Q}_{\geq 0}$ $\quad \sum m_i \in 1$.

T^n_{an} analytic torus (kind of "Stein manifold" in this topology.

$$\mathcal{O}(T^n_{an}) = \sum_{(i)\in\mathbb{Z}^n} c_{i_1\dots i_n} z_1^{i_1}\cdots z_n^{i_n}, \text{ with}$$

$$\sup |c_{i_1\dots i_n}|\, r_1^{i_1}\cdots r_n^{i_n} < \infty$$

$\forall\ r_1 > 0, \dots, r_n > 0.$

$T^n_{an} \to \mathbb{R}^n$ $\quad (z_1,\dots,z_n) \mapsto (\log|z_1|,\dots,\log|z_n|)$ The sup gives mult. semi-norm.

(Lift converges on any (...)|.

Get sheaf of fns. $\pi_*\mathcal{O}_{T_n}$ on $\mathbb{R}^n$

If $\bigcirc$ convex, ~~and~~ then no higher cohomology. If not, have higher cohomology.

general fibres are like "compact tori" (is nonarchimedean setup).

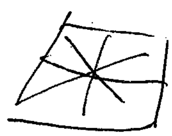
Generalized torus fibration:

X analytic space

$\downarrow$ proper

locally polyhedral.

$\leadsto$ discriminant locus



Def: $b \in B^{\text{good}}$ if $\exists$ open

$U_b \simeq U' \subset \mathbb{R}^n$ ~~std fibr~~ & identifications

$$\begin{array}{ccc} \text{pullback of } \underset{\simeq T^n_u}{(u)} & = & \pi^{-1}(U_b) \\ \downarrow & & \downarrow \pi \\ \mathbb{R}^n \supset U & \approx & U_b \end{array}$$

Get $GL(n, \mathbb{Z}) \ltimes \mathbb{R}^n$ ~~action~~ on good part.

Fact: if $B = \mathbb{R}^n = B^{\text{good}}$.

$\uparrow$

X,

then $X \simeq T_{an}$ unique up to multi by λ

Unlike non-Arch, Archimedean case has crazy automorphisms, ~~but~~ $(\mathbb{C}^\times)^2$

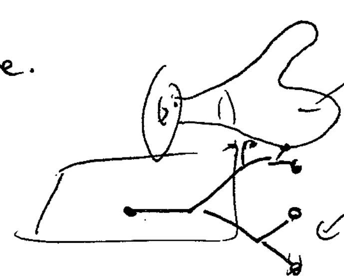
$(z_1, z_2) \to z_1, e^{z_1} z_2$

A-model.

Say X

$\downarrow$ manifold w/ $\mathbb{Z}$-affine structure.

$B \supset B'$ open dense good



hol. discs obstructs

affine tree

some ending in

disc locus

Proposal: could make generating fn. for # of trees,

& should dictate ~~gluing~~ change of coordinates

Observation: Say X Calabi-Yau, so smooth + carries vol_X n-form

(don't even assume non-degenerate) $\neq 0$.

on $\mathcal{B}'$, get $\mathbb{Z}$-affine function, something like

$\log |vol_X|$.

$z_1, \dots, z_n$ $\qquad \wedge\, \frac{\partial z_i}{\partial z_j}$

$\mathcal{B}' \supseteq \mathcal{B}''$ ← is max on which this function is constant.

Thm: Union of all images of all open discs ~~which~~ $\hookrightarrow X$

non-trivial

$\cap\, \mathcal{B}'' = \bigcup_{\text{countable}}$ hyperplanes.

Say have

$t = u_0$.

means

$\varphi^* vol_X$

$(1 \quad 1 = \text{const},$ $1-\varepsilon|t|e|$

$\bigwedge_1^{n-1} \frac{du_i}{u_i} \wedge \frac{dt}{t})$

$0 \leq |t| < 1$

$u_1, \dots, u_{n-1}$, and $\alpha_i < |u_i| < \beta_i$.

Near $1-\varepsilon < |t| < 1$, $\left| \text{Jacobian } \frac{d\log z_i}{d\log u_i} \right| = 1$.

$\varphi^* vol_X = f \frac{du_i}{u_i} \dots dt$

no poles

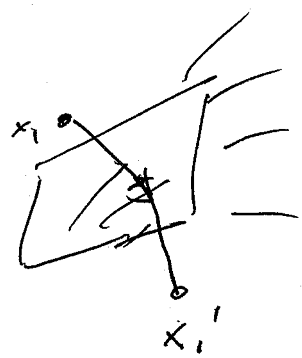
fix u_i, get fn.

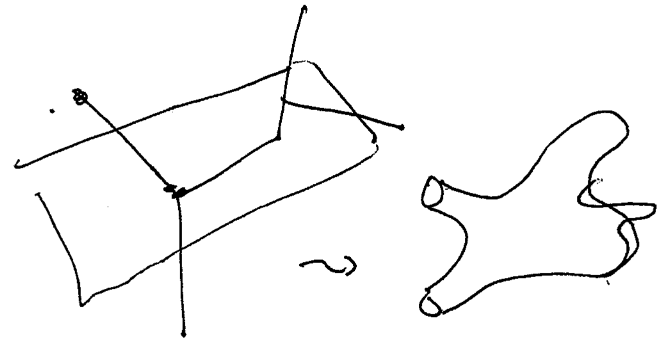
$\tilde{\varphi}(t)$ no poles on unit disc; looks like

$|e(t)| = \frac{1}{|t|}$ near boundary. By residue theorem, it's impossible.

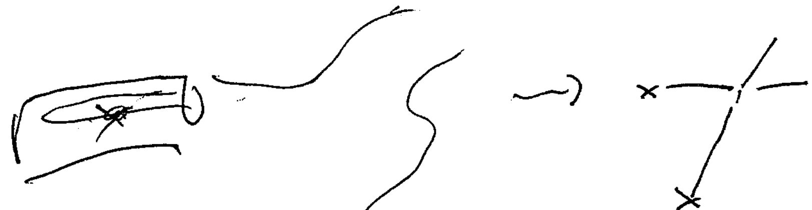
In particular, get existence of walls. Outside of walls, should get well-defined Fukaya category.

Idea: Not count discs at all, but ~~construct~~ change of coords. directly;





want to count cylinders w/ given directions



count $\mathbb{P}^1$'s.

degree of image in $\mathbb{R}^2$ is $n+1$.

Cap by gluing something in the ambient space.

(have no notion of real boundary, b/c cylinder form ∞-dim'l space.)

(when glue, have parameter space, its connected

(F. Pose : finite characteristic)

& min. intersection # invt. under deformation)

Constraint, like passing through some point, gives 0-dim'l

Counts have no boundary, b/c $\mathbb{P}^1$'s can only live in walls ??

Use usual GW invariants; but for just walls.

To get well-defined wall-crossing structures

B-model (+ Y. Soibelman)

Consider X: $z_1 z_2 = 1 + z_3$, $z_3 \neq 0$,
w/ $z_3 z_4 = 1$.

$\leadsto$ get $X^{an} \longrightarrow \mathbb{R}^3$

$(z_1, z_2, z_3) \longmapsto (\max(0, \log|z_1|), \max(0, \log|z_2|), \log|z_3|)$

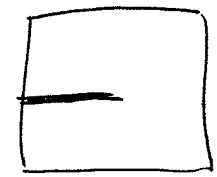
image is $\overset{\sim}{\text{homeo.}}$ to $\mathbb{R}^2$, so can project.

$$X^{an} \longrightarrow \mathbb{R}^2 \supset (\mathbb{R}^2 \setminus 0) = \mathbb{B}'$$

good part

$X^{an} \supset T^2_{z_1 \neq 0,\, z_3 \neq 0}$

$\supset T^2_{z_2 \neq 0,\, z_3 \neq 0} \xrightarrow{\log} \mathbb{R}^2$



get

Different maps & pop. of singularities $\longrightarrow \mathbb{R}^2$

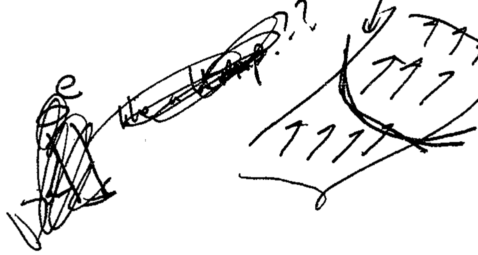
Can move.

General, $z_1 z_2 = \prod_{i=1}^{k} (z_3 + \lambda_i)$

$|\lambda_i| = 1$.

Say $B = \mathbb{R}^n \supset \mathbb{Z}^n$.

hypersurface transverse to vector field.



rat'l hyperplane constant vector field w/ rat'l slope (cly. vector & covector w/ pairing 0),

Mutations: F-Zelevinsky,

& canonical analytic space invariant under mutations.

vectors hyperplanes, single tropic + polynomial in one variable.

Global fns. on this space w/ polynomial growth at ∞

$|\mathcal{O}(-)|$ algebra. cluster algebra (generalization, don't need sympl. structure

defm, get a notion of mutations

If $\mathbb{Z}^n$ has skew form, recovers cluster

(Throw away fns. at ∞, see fns embedded in space),