Neitzke, Open-string Mirror Symmetry + Hitchin's Eq.

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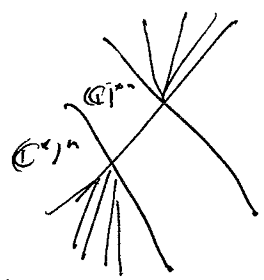
Today: I. Diagrams for Extended Scattering.

Recall ~~...~~ notion of wall-crossing structure / scattering diagram / K-wall network

B



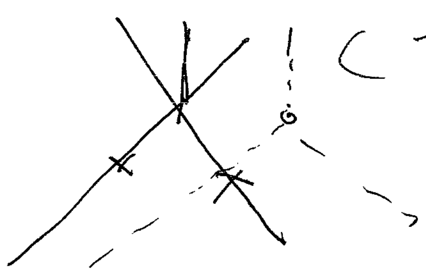
codim 1 walls

each wall carries birational map $(\mathbb{C}^\times)^n \to (\mathbb{C}^\times)^n$.

The diagram is a blueprint for building a $\mathbb{C}$-manifold.

(Kontsevich-Soibelman, Auroux, Gross-Siebert, Gross-Hacking-Keel

I'll describe an extension of these diagrams



(The walls of the extended diagram will carry standard action of fiducial torus w/ fiducial vector bdle.)

for building hol. (symplectic) cx. manifolds w/ a hol. vec. bdle

Uses:

1) By studying a (suitable) need to check asymptotics. $\mathbb{C}^{\times}$-family of such gluings, all ~~structures~~ structures upgraded: ~~...~~

M becomes hyperkähler!

E becomes a hyper-holomorphic

Def: A hyper-hol. vec. bdle on a HK manifold $\mathcal{M}$ w/ $\mathbb{C}$-strs. M_ζ ($\zeta \in \mathbb{CP}^1$) is a v.b. w/ a unitary connection, s.t. the curvature F_D is of type $(1,1)_\zeta$ for all $\zeta \in \mathbb{CP}^1$.

(analogue of a Yang-Mills instanton: in 4-D case, says F_D anti-self dual)

(same as hol. vec. bdle on twistor space, trivial on each fiber of $\mathbb{CP}^1$)

Dream: use extended diagrams to make explicit examples of HK metrics, HH bundles.

More likely: produce asymptotic formulas for them.

(2) The HH bundles we get often come in families:

Get a bundle over $M \times C^{\approx}$

$\uparrow$ param space

E.g. if C is cplx-1 dimensional, will find a solution to Hitchin equations.

Hitchin eq: For $E \longrightarrow C$, ~~equations~~ E $U(K)$ bundle w/ (unitary) connection D, & Higgs field $\varphi \in \Gamma(\mathrm{End}(E \otimes K_C))$.

obeying: $\bar{\partial}_D \varphi = 0$ covariantly holomorphic

$F_D + R[\varphi, \varphi^\dagger] = 0$ ($R \in \mathbb{R}_{>0}$ some scaling eq'n). (usual Hitchin eq'n: $R=1$)

together with restriction of diagram to curve C. $\leftarrow$ called a spectral network.

(Rmk: of course, $\mathcal{M}$ naturally fibred over $\mathcal{B}$),

Data: Hyperkähler integrable system data:

- $\mathcal{B}$ $\mathbb{C}$-mfold, $\mathcal{B}'$ complement of a divisor $\mathcal{B}$,
- Γ local system of lattices over $\mathcal{B}'$,

$0 \to \Gamma^f \to \Gamma \to \Gamma^g \to 0$ Γ extension

w/ $\langle\, , \rangle$ non-degenerate on Γ^g, pulled back to Γ.

- $Z : \Gamma \to \mathbb{C}$ (period, or central charge ?)

fibrewise homomorphism, holomorphic — i.e. $\forall$ $\gamma \in \Gamma$ local sections,

Z_γ holom. on B'.

- obeying conditions: dZ Γ^*-valued 1-form, vanishes on Γ_f.

i.e.: ~~descends~~ $(\Gamma^g)^*$ valued 1-form, w/

$0 = \langle dZ \wedge dZ \rangle$

and $\langle dZ \wedge d\bar{Z} \rangle > 0$ on B'.

- Scattering diagram: collection of codim.-1 walls on B, with the following properties: for each $\zeta \in \mathbb{C}^\times$

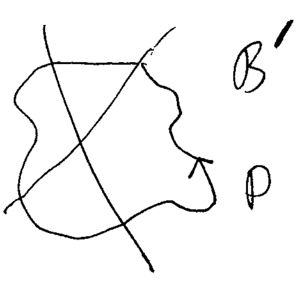
  - each wall carries a label $\gamma \in \Gamma$ section
  - a wall carrying a label γ lies in $\{ Z_\gamma / \zeta \in \mathbb{R}_{>0} \}$ (real codim-1 condition).
  - Define torus $T = \mathrm{Hom}(\Gamma^g, \mathbb{C}^*)$.
  - Wall w/ label γ carries an automorphism of T,

of the form $X_u : T \to \mathbb{C}^\times$

$$X_u \to X_u (1 - X_\gamma)^{\langle \gamma, u \rangle \Omega(\gamma)}$$

evaluation map $[K-S]$. (can interpret as D-T inv. but have just thought of as #; attached to walls.)

Constraint:



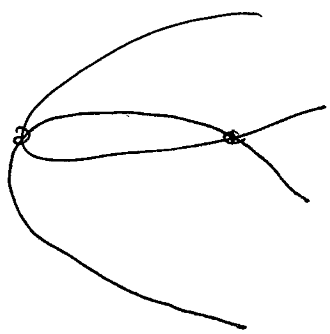
$\prod$ of automorphisms we meet going around p should be 1) $\mathbb{1}$ if p contractible in B

2) inverse of the monodromy if B is not contractible.

...

$$\begin{array}{c} E \\ \big\downarrow \\ M \end{array}$$

quadratic bundle



E, W = fibrewise quadratic

$\rightsquigarrow$

$b \in H^2(E, \mathbb{Z}/2)$ twisting cocycle.

$$\mathcal{F}(M, b) \longrightarrow \mathcal{F}(E, W, b \otimes b).$$

$$L \longrightarrow D \searrow L.$$

map in $M \times E$

not full-faithful functor.