P. Pandit (missed beginning of talk; insert motivating diagram).

Def (Bridgeland): A stab. condition/str. on $\mathcal{C}$ is given by

- $\{\mathcal{C}_\theta \subseteq \mathcal{C}\}_{\theta \in \mathbb{R}}$ s.s. objects of phase θ
- $Z : K_0(\mathcal{C}) \to \mathbb{C}$ (central charge) such that axioms hold s.t.

(1) $\mathcal{C}_{\theta+\pi} = \mathcal{C}_\theta[1]$

(2) $\hom(\mathcal{C}_\theta, \mathcal{C}_\phi) \simeq 0$ if $\theta > \phi$.

(3) $Z(\mathcal{C}_\theta) \subseteq \mathbb{R} e^{i\theta}$.

(4) (Harder–Narasimhan filtration): $\forall\, \mathcal{E} \in \mathrm{ob}\,\mathcal{C}$, there exists a diagram

$$0 \to \mathcal{E}_0 \to \mathcal{E}_1 \to \cdots \to \mathcal{E}_n \simeq \mathcal{E}$$

s.t. $\mathrm{cofib}(\mathcal{E}_i \to \mathcal{E}_{i+1}) \in \mathcal{C}_{\theta_i}$ with $\theta_1 > \cdots > \theta_n$.

How to construct Z?

Say have $\{F_v : \mathcal{C} \to \mathrm{perf}(k)\}_{v \in A}$ (family of functors), $z_v \in \mathbb{C}$ (complex #'s).

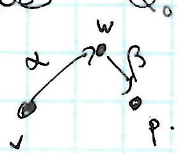
can define $\rightsquigarrow Z(E) = \sum_{v \in A} z_v\, \chi(F_v(E))$.

& get a line bundle L_v on $\mathcal{M}_{\mathcal{C}}^{ps}$, $L_v|_E = \det(F_v(E))$.

Fact: $\sum_v -c_1(L_v)\, \mathrm{Im}(e^{i\theta} z_v) \geq 0$ (pos. (1,1) form) on $\mathcal{M}_{\mathcal{C}_\theta}^{ps}$

(the sum is a Kähler class.)

Q Quiver, Q_0 - vertices, Q_1 - arrows



$\mathcal{C} = \mathrm{Rep}(Q, \mathrm{perf}(k))$

$\mathcal{E}_v^\bullet \xrightarrow{T_\alpha} \mathcal{E}_w^\bullet$

$\mathcal{E} = \oplus \mathcal{E}_v$.

For $v \in Q_0$, $F_v(\mathcal{E}^\bullet) = \mathcal{E}_v$.

given $z_v \in \mathbb{H}$,

$$\rightsquigarrow Z : K_0(\mathrm{Rep}(Q, \mathrm{perf}\, k)) \longrightarrow \mathbb{C} \quad (*)$$

Thm [King]: there is a stability condition on $\mathcal{C} = \mathrm{Rep}(Q)$ with this central charge $(*)$

& $\mathcal{C}_\theta = \{ E \mid E \in \mathrm{Rep}(Q, \mathrm{Vect}_k)$ concentrated in one degree & for all $F \subseteq E$, $\arg Z(F) < \arg Z(E) \}$ for $\theta \in [0, \pi)$.

$E \in \mathcal{C}_\theta$ - s.s. objects of slope θ

E is polystable if it is semi-simple ($\simeq \oplus$ simples) in $\mathcal{C}_\theta$

[King] E ss polystable iff ∃ a hermitian metric on E s.t.

$$\mathrm{Arg}(\mathbb{P}) = \theta\, \mathrm{Id}.$$

↖ e.g., w.r.t. polar decomposition (unitary part).

$$P = \sum_{v \in Q_0} z_v P_v^v + \sum_{\alpha \in Q_1} [T_\alpha^*, T_\alpha].$$

$G = K_{\mathbb{C}}$ ↻ induced action of complexification (doesn't preserve ω).

General setup: $(X, J, \omega^{1,1}) \subseteq \mathbb{P}(V)$

K ↻ cpct. group acts by kähler isometries

$\Phi : X \xrightarrow{\#} \mathfrak{k}^\vee$ moment map.

Thm (Kempf-Ness)

The GIT quotient (algebraic geometry)

$$X^{ps}/G \cong \text{sympl. quotient } \Phi^{-1}(0)/K$$

↑ polystable rep.

Infinite-dimension analogue:

Thm (Donaldson-Uhlenbeck-Yau): {Polystable bundles on Y} ←1:1→ {Hermitian Yang-Mills connections on Y}

Y a kähler manifold. ("polystable bdles" ⟷ those which contain a HYM representative)

⟹ ∃ a kähler metric on polystable bundles.

to ~~generalize~~ get B-model version of DUY: replace "B-branes" (complexes of ~~bundles~~)

bundles ⇝ complexes of bundles

HYM ⇝ deformed HYM eq'n.

Big conjecture: (Yau, Thomas, Kontsevich, Joyce ...)

(X, ω) symp. manifold, say part of kähler str. (X, ω, J)

Ω hol. volume form, then ∃ a stability structure on ~~[crossed out]~~ $\mathcal{C} \in \mathrm{Fuk}(X, \omega)$

s.t. $\mathcal{C}_\theta = \{L \subseteq X \text{ s.t. } \arg \Omega|_L = \theta\}$.

$Fuk(X,\omega)$ is defined $/ \mathbb{C}((t^{\mathbb{R}}))$ non-Arch. field.

Choosing a Lag'n L representing an A-brane $\langle L \rangle \iff$ choosing a metric on L,

Given $\mathcal{C}$, a K-linear category

A category of metrized objects for $\mathcal{C}$ is a functor $\mathcal{C}^{o} \to \mathcal{C}$

The space of metrics of E.

$$\begin{array}{ccc} Met(E) & \longrightarrow & \mathcal{C}^{o} \\ \downarrow & \ulcorner & \downarrow \\ \{E\} & \longrightarrow & \mathcal{C}^{\cong} \end{array}$$

Over $\mathbb{C}$, examples: $\mathcal{C} = Rep(Q)$, $\mathcal{C}^{o}$ = representations w/ Hermitian metrics.

Over K non-archimedean: $\mathcal{O}_K \subseteq K$ (ring of integers), residue field k.

Def'n: $\mathcal{C}^{o}$ is any category w/ an equivalence $\mathcal{C}^{o} \otimes_{\mathcal{O}_K} K \simeq \mathcal{C}$.

Setup: $\mathcal{C}/_K$, $\mathcal{C}^{o}/\mathcal{O}_K$, $\mathcal{C}_{sp} = \mathcal{C}^{o} \otimes_{\mathcal{O}_K} k$,

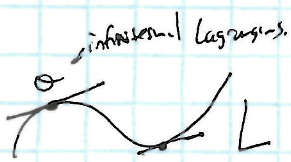
and given a stability condition on $\mathcal{C}_{sp}$.

Def: A metrized object $\mathcal{E} \in \mathcal{C}^{o}$ is θ-harmonic if $\mathcal{E} \otimes_{\mathcal{O}_K} k$ is polystable of phase θ in $\mathcal{C}_{sp}$.

"Baby categorical DUY."

Thm: (Haiden–Katzarkov–Kontsevich–Pandit): Given the setup, $\exists$ a stability structure on $\mathcal{C}$ (generic fiber), such that $E \in \mathcal{C}^{ps}_{\theta}$ if and only if $\exists$ a metrization $E' \in \mathcal{C}^{o}$, + an isomorphism $E' \otimes_{\mathcal{O}_K} K \xrightarrow{\sim} E$, such that E' is harmonic.

(X, ω) Ω



"semi-classical Lagrangian"

Finite dimensional context:

Flow: $-\text{grad} \|\Phi\|^2$

Fixed points: "harmonic representatives."

moment map $\|\Phi\|^2$

$\psi_x : G/_K \to \mathbb{R}$ "space of metrics" (ex. $G = GL_n$, $K = SU_n$)

$\psi_x(gx) = \frac{1}{2} \log \|gx\|^2$ and $\psi = \Phi^{-1}(0)$.

∞-dim'l example

YM flow on connections, heat flow on metrics.

HYM-connections

YM-functional, "secondary characteristic classes"

Categorical framework:

~~Def~~ (1) A flow on the space $Met(E)$

(2) $M: Met(E) \longrightarrow \mathbb{R}_{\geq 0}$ $\quad M \searrow F \quad$ mass

(3) $Z: K_0(\mathcal{C}) \longrightarrow \mathbb{C}$ $\quad M \geq |Z|$ BPS inequality

(4) $S_{\mathbb{C}} : ob\, \mathcal{C}^{\circ} \longrightarrow \mathbb{C}$

plurisubharmonic ($\Rightarrow$ kähler potential!)

A-model: A metrization is a choice of L representing the Ham. isotopy class. (determining object of $\mathcal{E}^{\circ}$).

Tangent space to Ham. isotopy class of L = space of exact 1-forms df.

- Flow on $L = d \operatorname{Arg} \Omega|_L$.
- Mass: $\int_L |\Omega_L|$
- Z: $\int_L \Omega$
- $dS_{\mathbb{C}}(df) = \int_L f \Omega_L$.

Quivers: $P = \cdot \rightarrow$

Mass = $Tr(P) = Tr(\rightarrow)$

central charge $= \overline{}$

$S_{\mathbb{C}}^{(h_0,h)} = \log \det(H)$, where $H = h_0^{-1} h$?