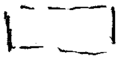
V. Shende   Mirror symmetry:

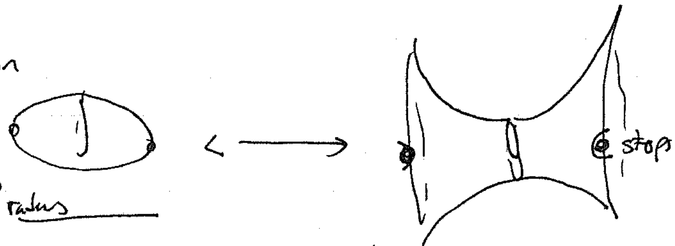
Coh		Fuk
$\bullet$	$\longleftrightarrow$	$\bullet$
$\mathbb{C}^*$	$\longleftrightarrow$	T^*S^1
$\mathbb{P}^1$	$\longleftrightarrow$	?

(or stabilize by "$\times T^*\mathbb{R}$").



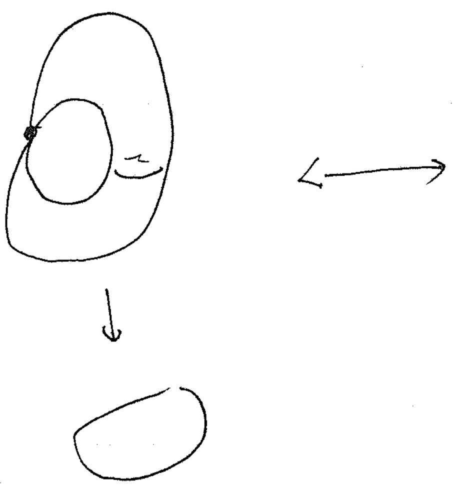
circle fibration

shrinking to 0 radius

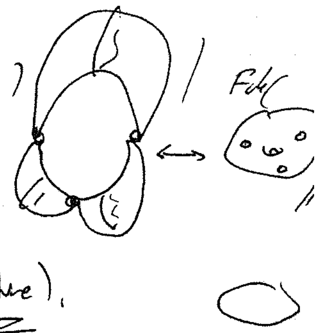


stops.

blow up to ∞ radius (so can't "wrap" all the way around") $\longleftrightarrow$ stops.



Coh($\mathbb{C}^*$) OR:

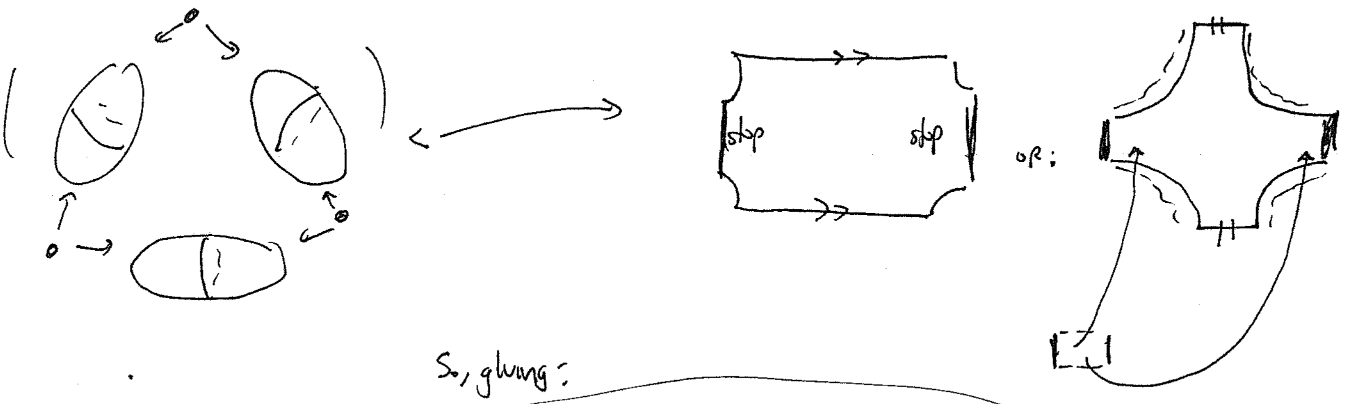


/ Fuk(

lives over singular point. (puncture). (rather than stop).

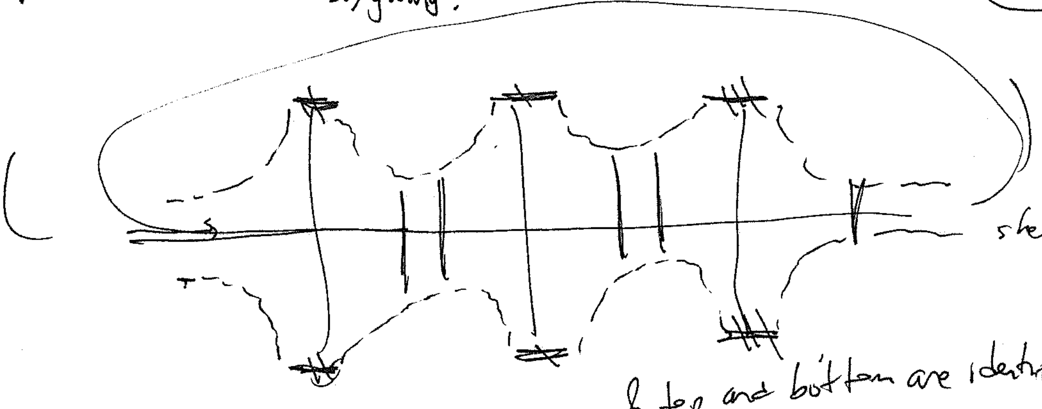
Can try to make (*) by gluing:

Coh



So, gluing:

Fuk(



skeleton = 3 circles meeting along a circle

& top and bottom are identified

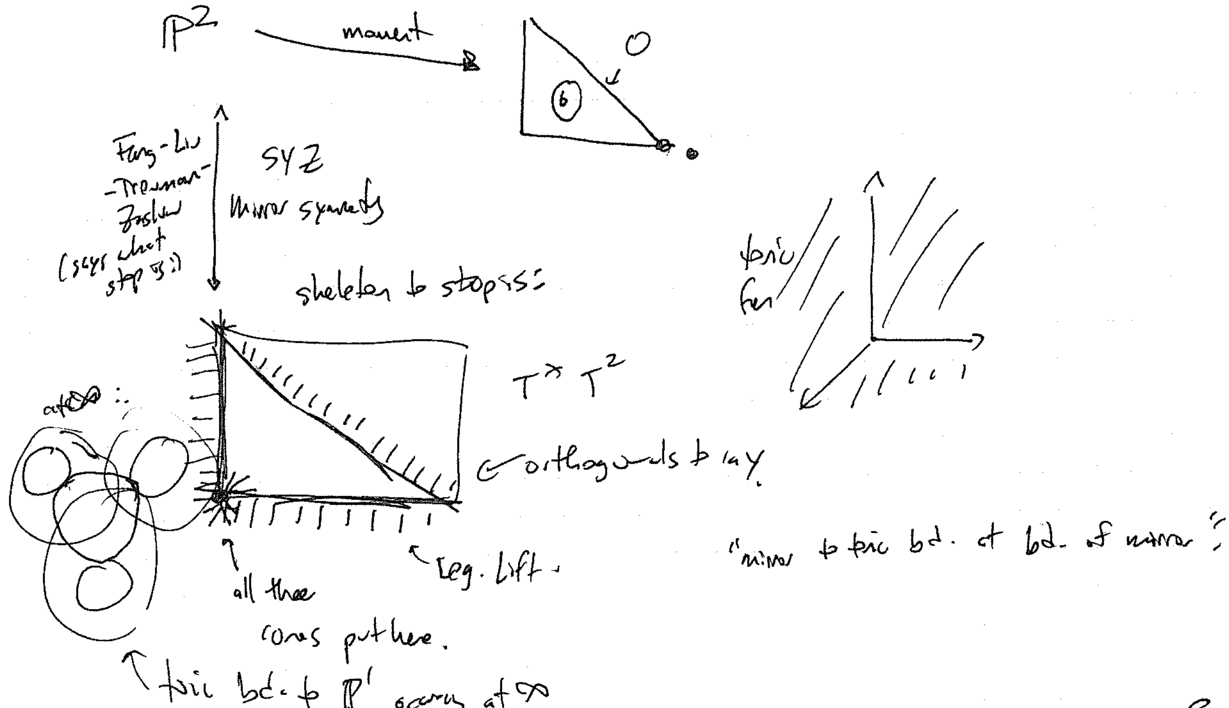
Kontsevich's localization conjecture

(reformulation):

Given a Liouville domain W & a cover $W = \bigcup W_i$ by Liouville sectors,

then $\mathrm{WFuk}(W) = (\text{ho})\mathrm{colim}\left(\mathrm{WFuk}(W_i) \leftleftarrows \mathrm{WFuk}(W_i \cap W_j) \leftleftarrows \quad \right)$.

Higher dimensions:



Thm (Gammage-S.): Let Π be any smooth toric stack, & $f : (\mathbb{C}^*)^n \to \mathbb{C}$

a generic polynomial whose Newton polytope is Δ_Π. Then, (can choose coords s.t.)

the relative skeleton of f is the FLTZ skeleton for Π.

(in other words the rel. skeleton of the LG mirror, e.g.

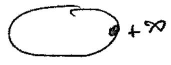
$W = x + y + \frac{1}{xy}$ agrees w/ FLTZ skeleton).

what's the rel. skeleton? $\mathbb{P}^2$:

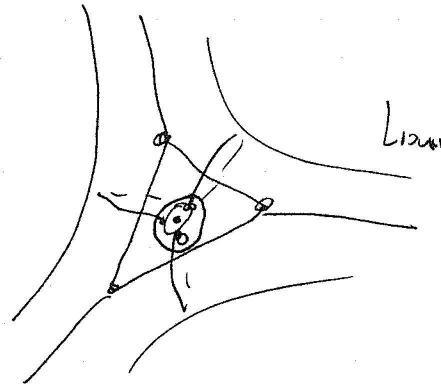
Def: (Rel. skeleton): (W, W') Liouville pair (e.g. $W' \subset \partial^\infty W$ Liouville hypersurface)

then $\mathbb{L}_{(W,W')}$

Given $X \xrightarrow{f} \mathbb{C}$, $(X, f^{-1}(+\infty))$ is a Liouville pair, so $\mathbb{L}_f := \mathbb{L}_{(X, f^{-1}(+\infty))}$.



Pf:



Liouville str. w/ potential $|\log d(0,-)|^2$.

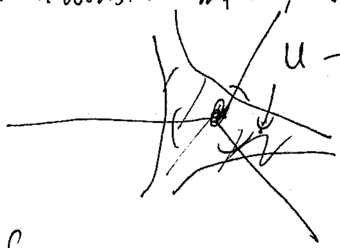
Since Morse fxn. on ambient space is radial, it's just the cone.

Cor: $\partial \amalg_f$ admits a cover by the corresponding mirror skeleta of components of the toric boundary.

Cor: Any sufficiently functorial mirror symmetry for toric varieties + localization conjecture. $\Rightarrow$ mirror symmetry for very affine hypersurfaces. (e.g., hypersurfaces in $(\mathbb{C}^\times)^n$).

Localization

~~Thm in progress~~ Thm (Ganatra-Pardon-S): Given a Weinstein mfld, $\exists$ a presheaf of categories on its skeleton,



$U \to$ Fukaya cat. of sector corresp. to it.

Thm in progress

This presheaf is a ~~sheaf~~ cosheaf.

$$\operatorname{colim} \mathcal{C}_\alpha = \underbrace{\text{oplax colim } \mathcal{C}_\alpha}_{\text{what's this?}} / \sim$$

$$\text{oplax colim}\left(\begin{tikzcd} E & & D \\ & C \arrow[ul, "g"] \arrow[ur, "f"'] & \end{tikzcd}\right)$$

objects $= \amalg$ C, D, E

morphisms same if objects both in C, D, E

but do exist morphisms

$$\begin{array}{cc} C & D \\ \downarrow & \downarrow \\ c & \operatorname{Mor}(f(c) \to d) = \operatorname{Mor}(c, d) \end{array}$$

& $g(c) \longrightarrow e \in E$ $= \operatorname{Mor}(c, e)$

?

$$\operatorname{colim}\left(\begin{tikzcd} E & & D \\ & C \arrow[ul] \arrow[ur] & \end{tikzcd}\right)$$

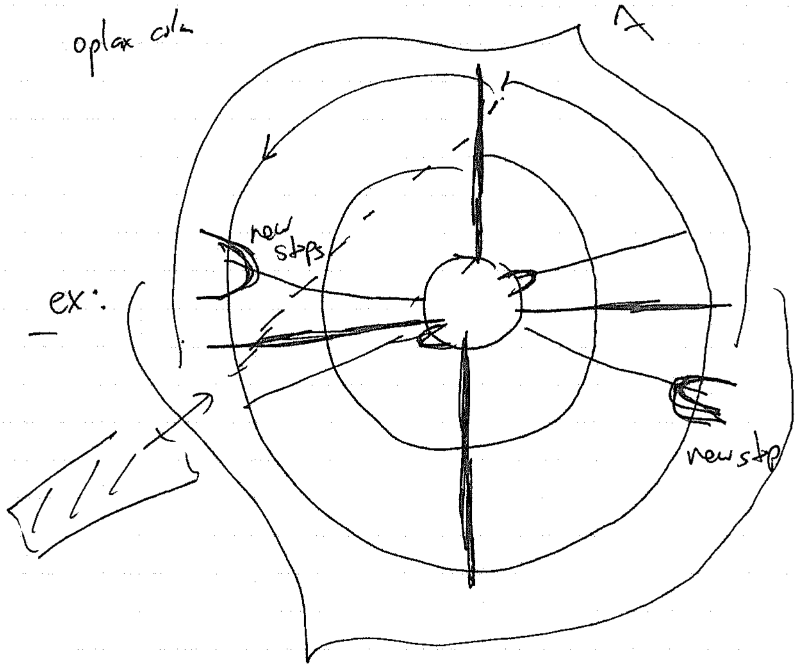
invert ~~$f(c) \xrightarrow{1} f(c)$~~

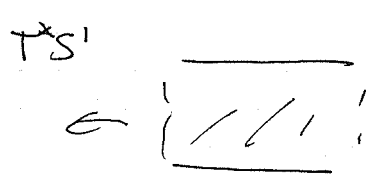
$c \longrightarrow f(c)$

induced by $\mathrm{id}_{f(c)}$;

oplax colim

ex:





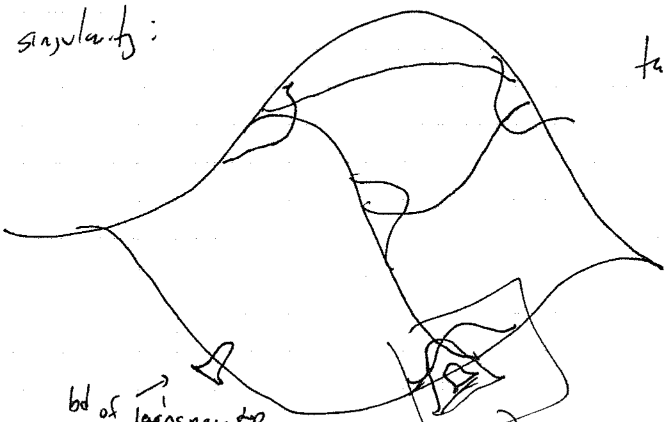
this gives us the oplax colimit.

B.

to invert to get to the genuine colimit, kill some objects (e.g., the dotted object) remove stops, & its the wrapped category.

In higher dimensions:

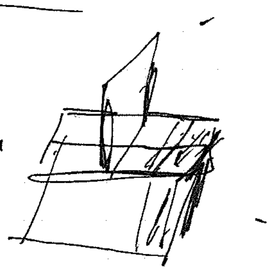
A_3 arbored singularity:



+ explain why stops get removed ~~stps~~ when invert.

take this as a stop

skeleton



inclusion of a subsector

add stops to capture data

take care (cancel overlap)