Day 5 Talk 1 - Lino,

<u>A_n singularities</u>

($\underline{\underline{\dim = 2}}$)

$$x^2+y^2+z^{n+1}=0 \quad n \geq 2.$$

$$\downarrow$$

$$p_w(z) = z^{n+1} + w_n z^n + \cdots + w_0$$

$g_w = x^2+y^2+p_w(z)$, $w=(w_0,\dots,w_n)$ multi-index

$\boxed{g_w = 0}$ $\quad \omega_{\mathbb{C}^3}$, $\Theta_{\mathbb{C}^3}$

$c_1 = 0$, so we can grade.

$$\eta \wedge dg = dz_1 \wedge dz_2 \wedge dz_3$$

$\bar{w} \in W \subseteq B^{2n+2}(\delta)$

$X = g_{\bar{w}}^{-1}(0) := E_{\bar{w}}$

s.t. E_w <u>smooth</u>

$E = \{(E_w, w)\}$

$\downarrow$

W

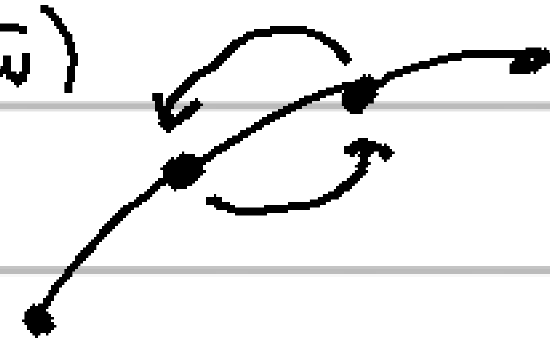
We're restricting to guys w s.t. No <u>Repeated</u> zeroes.

$E = \{(E_w, w)\}$

$W \hookrightarrow \mathrm{Conf}^{m+1}(D^2, \partial^2)$

$w \rightsquigarrow \{\text{zeroes } p_w\}.$

$$\begin{array}{l} \pi_1(\mathrm{conf}^{m+1}) = \pi_1(W, \bar{w}) \\ \quad \wr\!\wr \qquad\qquad\qquad \downarrow \\ \quad Br_{m+1} \qquad \text{``}\pi_0 \, \mathrm{Symp}(X)\text{''} \end{array}$$



So Braid group Br_{m+1} acts on X through symplectic automorphisms

<u>Thm</u>: $Br_{m+1} \xhookrightarrow{\rho} \pi_0(\mathrm{Symp}(X))$ injective.

(for D_n, E_m, e.g. map exists but not known whether it's injective.).

$\overline{\mathrm{Fuk}(X)}$:

$$\begin{array}{ccc} X & & \{x^2 + y^2 + p(z) = 0\} \\ \downarrow \pi & & \downarrow \\ \mathbb{C} & & z \end{array}$$

$\Delta = p^{-1}(0)$

Fix z_0, $\pi^{-1}(z_0) = Q_z$.

If $z \notin \Delta \Rightarrow Q_z \cong T^*S^1$.

Special case of this:

$$x^2+y^2=1.$$

$$\Big\downarrow$$

$$(u_1^2+v_1^2)-(u_2^2+v_2^2)=1$$

$$\langle u, v\rangle = 0.$$

$(u,v)\in(\mathbb{R}^2)^2$ $\quad \|u\|^2-\|v\|^2=1$

$$\langle u,v\rangle=0$$

$$\Big\downarrow$$

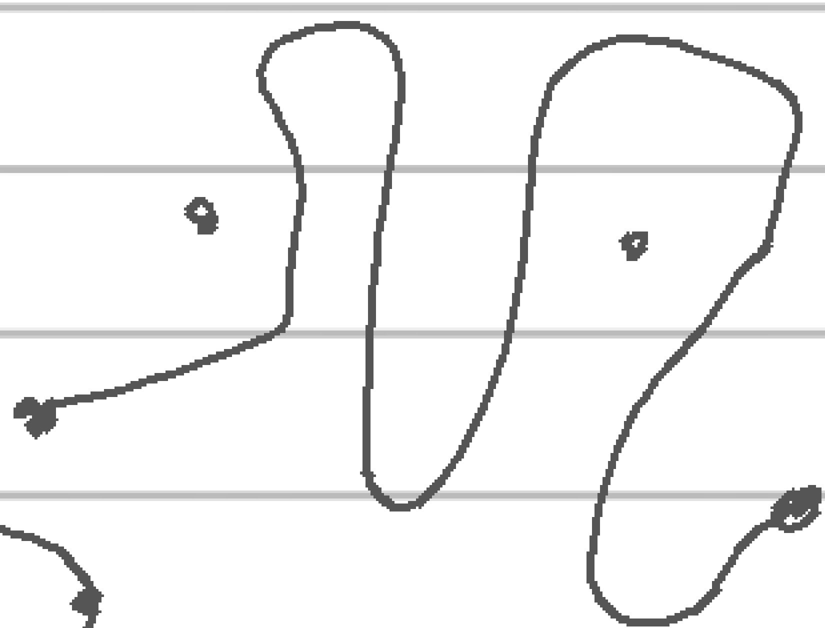
$$\left(\frac{u}{\|u\|},\ v\|u\|\right) \in T^*S^1 = \begin{cases}(u,v),\ \|u\|=1\\ \langle u,v\rangle=0.\end{cases}$$

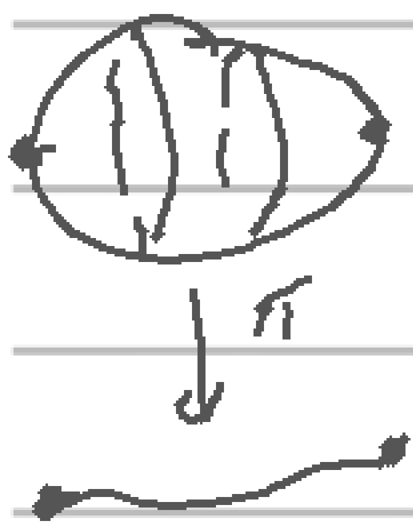
If $z\notin\Delta$, $Q_z = T^*S^1$, & $\Sigma_z = S^1$ $\quad$ Σ_z is zero section.

If $z\in\Delta$, $Q_z = T^*S^1/S^1$, $\Sigma_z = pt$.

$$L_\gamma = \bigcup_{z\in\gamma} \Sigma_z$$

Ex:





Prop: ① L_γ are Lag'n spheres

② $\gamma \overset{\text{iso}}{\sim} \gamma' \Rightarrow L_\gamma \overset{\text{ham. iso}}{\sim} L_{\gamma'}$

$$\operatorname{rk} HF(L_\gamma, L_{\gamma'}; \mathbb{Z}/2) = 2I(\gamma,\gamma')$$

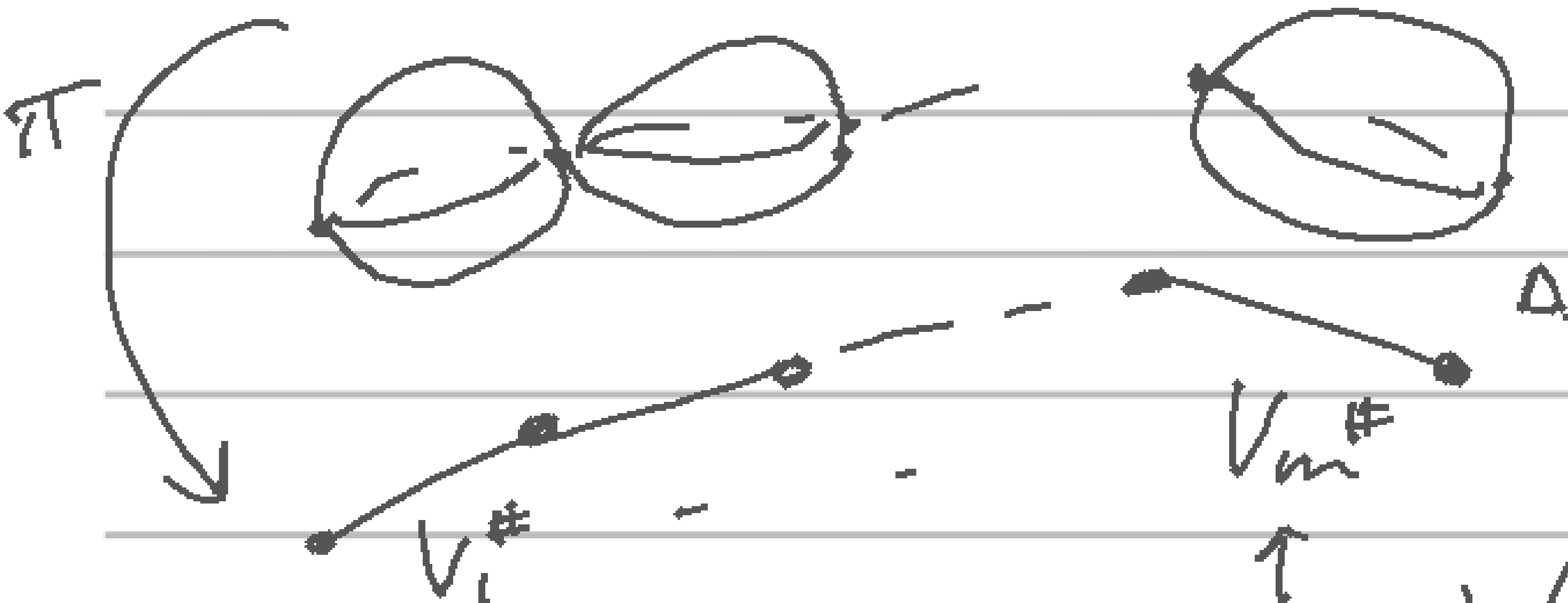
$$\sum_{z\in\gamma\cap\gamma'} |\gamma\cap\gamma'\setminus\Delta| + \frac{1}{2}|\gamma\cap\gamma'\cap\Delta|$$

endpts. count w/ weight 1/2.

Rmk: About transverse intersections: $L_\gamma, L_{\gamma'}$ ought to intersect cleanly @ endpoints, use Morse-Bott theory for middle intersections

Idea for counting intersections: first remove excess intersections of γ, γ'; use Morse-Bott type perturbations for rest.

$$X = \{x^2 + y^2 + p_{\vec{w}}(z) = 0\}.$$



equipped w/ choice of gradings.

Call $\mathcal{F}_m \langle (V_1^\#, \dots, V_m^\#) \rangle \subseteq Fuk(X)$ the full subcat. gen. by these objects.

Pick gradings so that intersections are

<u>A_m chain</u> <u>spheres</u>°

We can calculate $H^\circ(F_m)$ formally

(Paul: total space deformation retracts onto these spheres, i.e. $\vee S^2_i$).

$$HF^*(V_i^\#, V_i^\#) \overset{PSS}{\cong} H^*(S^2) \text{ b/c}$$

everything is exact

$$HF^*(V_i^\#, V_j^\#) = \begin{cases} 0, & |i-j| \geq 2. \\ \mathbb{Z}, & j = i \pm 1. \end{cases}$$

Products: Most should be zero.

$$\begin{tikzcd} V_1^\# \arrow[r, bend right, "\beta_i"', "0"] & V_2^\# \arrow[l, bend right, "\alpha_i"', "2"] \arrow[r, bend right, "0"] & V_3^\# \arrow[l, bend right, "2"] \end{tikzcd}$$

Use Floer Poincaré duality: (which works here!)

(i.e. $HF^d(L, L') \otimes HF^{n-d}(L', L) \to HF^n(L,L)$ non-degenerate
$$HF^n(L,L) \cong H^n(L) \cong \mathbb{Z}.)$$

so: $\alpha_i \beta_i = k_i \beta_{i-1} \alpha_{i-1}$.

$k_i \in \mathbb{C}^*$

Now multiply generators by some constant so each $k_i = 1$.

<u>Get A_m quiver</u>:

$$\begin{tikzcd} \bullet \arrow[r, bend right, "0"'] & \bullet \arrow[l, bend right, "2"'] \arrow[r, bend right, "0"'] & \bullet \arrow[l, bend right, "2"'] \arrow[r, bend right, "0"'] & \bullet \arrow[l, bend right, "2"'] \end{tikzcd}$$

<u>path algebra</u> (elts. are paths, products are composing paths).

modulo:

$$(i|i+1|i+2) = 0$$

$$(i|i-1|i-2) = 0$$

$$(i|i-1|i) - (i|i+1|i)$$

$$=: A_m$$

think of A_m as an algebra over $k = \mathbb{C}^m$

paths of length 0 correspond to id, paths of length 1 correspond to intersections, back & forth relations correspond to Poincaré duality argument.

① $V_i^{\#}$ split generate

② this algebra is intrinsically formal.

in particular, any A_∞ algebra that has homology this guy is formal, so we understand $\mathcal{F}(X)$ completely.

$\Longrightarrow \Pi \operatorname{Tw} \mathcal{F}(X) \simeq \operatorname{mod}(A_n)$

split closure

What is split closure? In a classical linear category, suppose you have an idempotent

$\pi : X \to X$, $\pi^2 = \pi$.

image π:

$$X \underset{i}{\overset{k}{\rightleftarrows}} Z \qquad k \circ i = id_Z \qquad i \circ k = \pi.$$

"$X = Z \oplus Z^\perp$"

We call the category split-closed if Z exist for any π.

(on A_∞ level, have to talk about idempotents up to homotopy). — won't define now

We'll use: A_∞ category A split-closed $\Longleftrightarrow$ $H^0(A)$ split closed.

(idea of Π is take the minimal split-closed enlargement).

$$A \hookrightarrow \prod A$$

minimal split-closed category containing A.

"Karoubi completion"

Why do these things split generate?

Related to $\tau_{V\#}(L) = T_{V\#}(L)$

We use the following algebraic theorem: If you have a family of spherical objects Y_i, s.t. $\forall X$,

$$T_{Y_1} T_{Y_2} \cdots T_{Y_m}(X) = X[\sigma]$$

$\neq 0$,

then $Y_1, \ldots, Y_m$ split generate.

So have to compute "global monodromy" of the fibrations. In general, not identity.

In this case, if

$$\phi = \tau_{V_1} \cdots \tau_{V_m}, \quad \neq 0$$

$$\phi^{2m+2}(L) = L[\sigma] \text{ for } L \text{ cpt. Lag'n } \subseteq K.$$

comes from fact that orig. poly was weighted homogeneous.

② Intrinsic Formality: Recall,

Thm: $HH^q(A_m, A_m[2-q]) = 0$, $q \geq 3$,

$\Rightarrow$ if $\mathcal{A}$ has $H(\mathcal{A}) = A_m$, $\mathcal{A}$ formal

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Now, just compute $HH^q(A_m, A_m[2-q])$.

Doesn't use any big theorems, just write it down.

(Paul: secretly relies on some geometric intuition).

This is much easier $\forall$ $n > 2$, but in dim 1, other things happen.

(The analogue of $QH \to HH$ is an iso. here).
