Day 4 Homework:

Directed A_∞ category A_n

$$\underbrace{\bullet \to \bullet \to \bullet \to \cdots \bullet}_{n}$$

all compositions are 0.

Problem 1: Take A_3 & consider the following twisted complexes:

$$\left.\begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix}\right\} \text{basic objects}$$

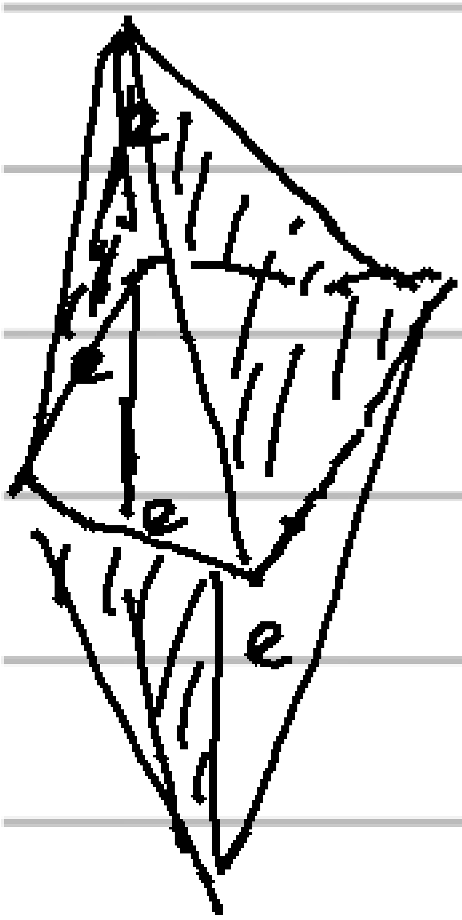
$\mathrm{Cone}(X_1 \to X_2)$ $\quad$ $(X_1 \xrightarrow{\delta} X_2 \xrightarrow{\delta} X_3)$

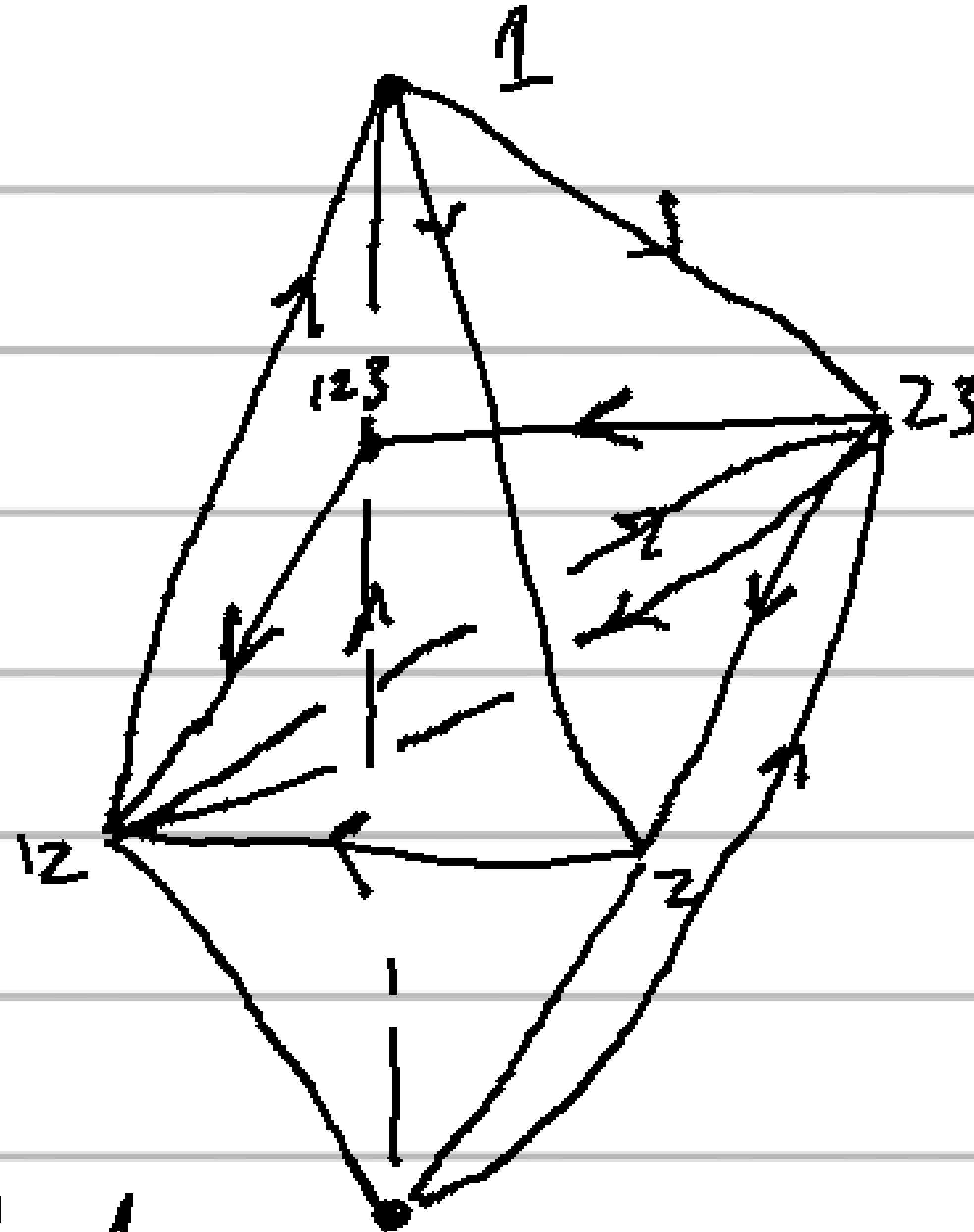
$\mathrm{Cone}(X_2 \to X_3)$

Draw the full subcategory of $H(\mathrm{Tw}\,A)$ consisting of those 6 objects & which are the exact triangles between them?

Problem 2: Study the mutations of A_3 (if too boring, of A_4).

5 PM: <u>Answer</u>:





any $A_3 \longrightarrow \mathrm{Tw}\mathcal{A}$ extends to

$\mathrm{Tw}\,A_3 \longrightarrow \mathrm{Tw}\mathcal{A}$, giving us octahedral axiom.

higher order analogue of octahedral: replace A_3 w/ A_4.

Mutations of A_3: straightforward.

what's the Lefschetz fibration giving rise to A_3?

$\pi(x) =$ generic polynomial of degree 4

$\pi : \mathbb{C} \to \mathbb{C}$.

fiber $\rightsquigarrow$ $\underset{1}{\bullet} \quad \underset{2}{\bullet} \quad \underset{3}{\bullet} \quad \underset{4}{\bullet}$

(depends on choice of paths)

$L_1 = 12, \quad L_2 = 23, \quad L_3 = 34$

$$F^{\to} = (\bullet \longrightarrow \bullet \longrightarrow \bullet) = A_3.$$

clear that you mutate all you want, only get finitely many things.