Sarkar I:

Cohen, Jones, Segal

Floer's ∞-dim'l Morse theory & htpy type.

<u>Pontryagin</u> - <u>Thom construction</u>: (c.f.'s Milnor's <u>Differential Topology</u> book).

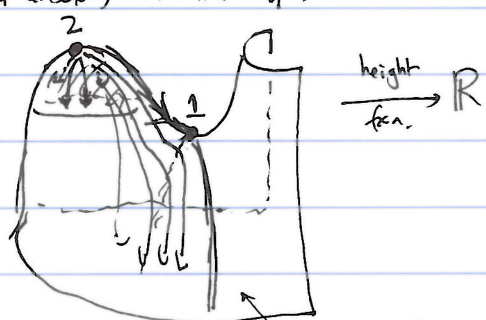
· Morse fns. w/ only 2 critical points. $\qquad$ <u>CW cplx</u> w/ 2 (non-basepoint) cells.

+ <u>generic gradient flow</u>

all descending + ascending disks intersect $\pitchfork$.

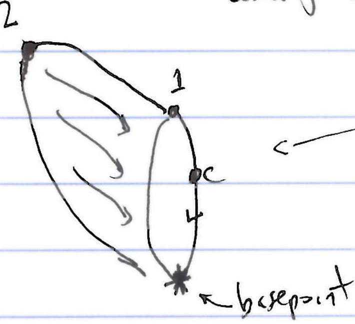
e.g., $M=$ (non-compact)

(a)



$\rightsquigarrow$ Union of all descending discs, and collapse everything below a certain height to a basepoint.

(b)



the cells corresp. to critical points, dim = index of crit. point

subtlety: parametrizations in (a) & (b) are <u>different</u>!

e.g., in (a), no points near 2 flow to the point c!

Say have index n, $n+k+1$ critical points

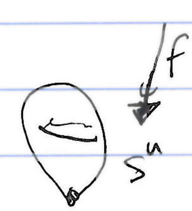
$\rightsquigarrow$ 2 cells of dimension n, $n+k+1$.

e^{n+k+1}

$\partial e = S^{n+k}$

e.g., need to specify $[f] \in \pi_{n+k}(S^n)$.

$f: S^{n+k} \to S^n$. based maps.



P-T:

maps $S^{n+k} \xrightarrow{f} S^n$ $\longleftrightarrow$ Framed k-dim'l manifold Z in $\mathbb{R}^{n+k}$

meaning normal bundle $\nu(Z)$ is trivialized.

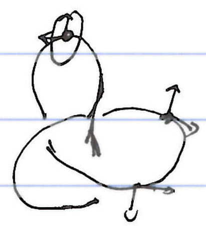
f, assume smooth (by perturbing) $\longmapsto$ preimage of regular value of f, $p \neq *$, & take $f^{-1}(p) \subseteq \mathbb{R}^{n+k} = S^{n+k} \setminus *$

k-dim'l manifold, framed b/c $T_p S^n$ is framed essentially canonically & preimage gets induced normal framing.

Using tub. nhood thm, $\longleftarrow$ Given $Z \subseteq \mathbb{R}^{n+k}$ framed.

given a framed manifold $Z \subset \mathbb{R}^{n+k}$, nhood $Z \subset U \cong Z \times \mathbb{R}^k$, $\pi_2: U \to \mathbb{R}^k$

Construct a based map $S^{n+k} \xrightarrow{f} S^n$ by:



$\subset \mathbb{R}^3$

$$f(x) = \begin{cases} * & \text{if } x \notin U \\ \pi_2(x) & \text{if } x \in U \end{cases} \quad \pi_2: U \to \mathbb{R}^k.$$

Dependence on choices: if chose a different regular value or choose homotopic f $\longmapsto$ changes $Z^k \subset \mathbb{R}^{n+k}$ by a framed cobordism in $\mathbb{R}^{n+k} \times I$.

& vice versa: change Z by cobordism $\longrightarrow$ maps f change by htpy.

(1) to go $\longleftarrow$, need a tubular nhood.

- if M was smooth, & normal bdle framed, this is easier.
- if M top-manifold, may not have), but maybe something else we can do! (c.f. Kupers talk)

(2) to go $\longmapsto$ need to approx. f by a smooth fcn, & apply Sard.

some analysis needed, right now

Morse fn.

2 critical points

$\mathrm{ind}(X) = n+k+1$

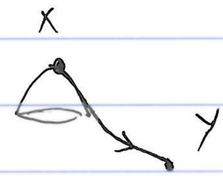
$\mathrm{ind}(Y) = n.$

CW cplx:

$S^{n+k} \xrightarrow{f} S^n$

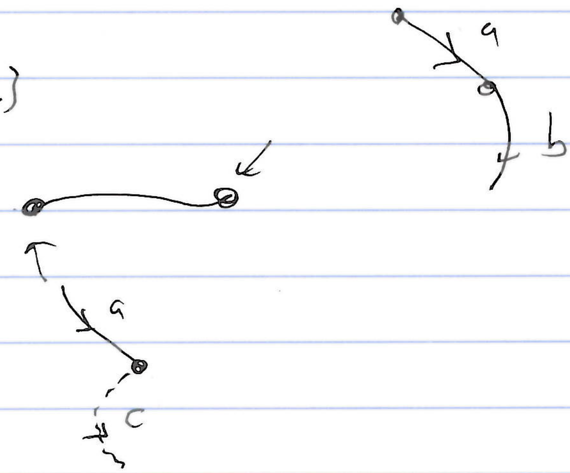
$f^{-1}(p)$ is a k-dim'l manifold

(e.g., choose $p = y$, then $f^{-1}(p) = \mathcal{M}(x,y)$).



$\mathcal{M}(x,y) = \{pt.\}$

$\mathcal{M}(x,z) =$ 



problem: need to keep track of all $\mathcal{M}(x,z)$ is higher dim'l etc as framed mflds w/ corners, etc.

references for manifolds w/ corners.

Jänich

Laures: On cobordisms of m'flds w/ corners.

A Flow category $\mathcal{C}$ has:

1) finitely many objects $Ob(\mathcal{C})$ (Morse critical points)

2) $gr : Ob(\mathcal{C}) \to \mathbb{Z}$ (index)

3) $\mathrm{Hom}(x,y) = \begin{cases} id & \text{if } x = y \\ \text{smooth m'fld of dim } k := gr(x) - gr(y) - 1 & \text{if } x \neq y \end{cases}$

$\mathcal{M}(x,y) \times \mathcal{M}(y,z) \to \mathcal{M}(x,z)$ smooth $\to \partial\mathcal{M}$

5) $\partial\mathcal{M}(x,z) \overset{\text{homeo.}}{\cong} \coprod_y \mathcal{M}(x,y) \times \mathcal{M}(y,z).$

$\mathcal{M}(x,y) \leftarrow$ "unparam'd cptified moduli of traj."

this + framing $\Rightarrow$ space-level.