Math 113 Midterm
Winter, 2010

Please print your name here:

This is a closed-book, no notes exam. Please sign here to indicate your acceptance of the Stanford Honor Code:

Unless otherwise indicated, you should prove each of your answers. You may use results proved in the text, in class, or in the problem sets, but you must clearly state the result before applying it to your problem. Your grade will reflect the quality of your exposition as well as the correctness of your argument.

Question	Score	Maximum
1		10
2		10
3		10
4		10
5		10
Total		50

1. (10 points)

   (a) State the Fundamental Theorem of Algebra.

   (b) Give an example of a linear transformation $T : V \to W$ that is not diagonalizable. (You do not need to prove your answer.)

2. (10 points) Let $\mathcal{E}(4)$ be the subspace of $\mathcal{P}(4)$ consisting of polynomials that are even functions. Show that there exists a subspace $U \subset \mathcal{P}(4)$ such that

$$\mathcal{P}(4) = \mathcal{E}(4) \oplus U.$$

(You do not need to verify that $\mathcal{E}(4)$ is a subspace. Furthermore, recall that $f : \mathbb{R} \to \mathbb{R}$ is even if $f(x) = f(-x)$ for all x.)

3. (10 points) Suppose that $T : V \to V$ has the property that every vector in V is an eigenvector for T. Show that T is a scalar multiple of the identity operator.

4. (10 points) Given an operator $T : V \to V$, show that T is injective if and only if it is surjective.

5. (10 points) Let $f, g \in \mathcal{L}(\mathcal{P}(1), \mathcal{P}(1))$ be the functions defined by the following equations:

$$f(1+x) = 2x + 1 \text{ and } f(1-x) = 1.$$

$$g(x) = x \text{ and } g(1) = 0.$$

(a) If $h \in \mathcal{L}(\mathcal{P}(1), \mathcal{P}(1))$ is defined by

$$h(1) = 2x + 2 \text{ and } h(x) = x,$$

is $h \in span\{f, g\}$?

(b) Extend $\{f, g\}$ to a basis for $\mathcal{L}(\mathcal{P}(1), \mathcal{P}(1))$.