# Math 171 Homework 1

Due Friday April 8, 2016 by 4 pm

Please remember to write down your name and Stanford ID number, and to staple your solutions. Solutions are due to the Course Assistant, Alex Zamorzaev, in his office, 380-380M (either hand your solutions directly to him or leave the solutions under his door).

**Book problems**: Solve Johnsonbaugh and Pfaffenberger, problems 5.4, 5.7, 9.10, 10.5 (give a formal proof of this fact from the definition of limit using the "Archimedean property of the reals" mentioned in Prof. Simon's notes), 13.2, 16.7, 16.10, 18.5, 20.13, and the following problems:

**1.** Let $\{a_n\}$ be a sequence of real numbers. We say a real number $\ell$ is a *cluster point* of $\{a_n\}$ if for any $\epsilon > 0$, there are infinitely many $a_n$ within $\epsilon$ of $\ell$. Formally, for any fixed $\epsilon > 0$, given any $N \in \mathbb{N}$, there exists an $n \geq N$ with $|a_n - \ell| < \epsilon$.

(a) Show that if $\ell$ is a cluster point of $\{a_n\}$, then there is a subsequence of $\{a_n\}$ converging to $\ell$.

(b) Suppose $\mathfrak{a} := \{a_n\}$ is a bounded sequence and let $C_{\mathfrak{a}}$ denote the set of all cluster points of $\mathfrak{a}$. Show that $C_{\mathfrak{a}}$ is bounded (so its sup and inf exist), and moreoever that

$$\limsup_n \{a_n\} = \sup C_{\mathfrak{a}}$$
$$\liminf_n \{a_n\} = \inf C_{\mathfrak{a}}$$

**2.** It may be used without proof that the complex numbers $\mathbb{C}$ satisfy the axioms of a field (axioms 1-11) in Johnsonbaugh and Pfaffenberger, or F1-F6 in Prof. Simon's notes. Prove that they *don't* satisfy the axioms of an ordered field (axiom 12 in JP or O1-O2 in Prof. Simon's notes).