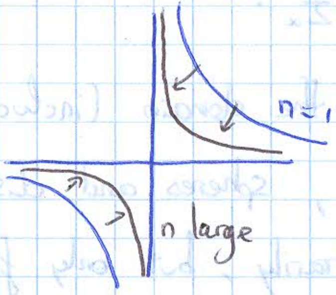
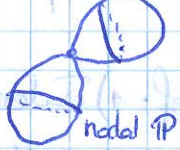
ex: $u_n : S^2 = \mathbb{CP}^1 = \mathbb{C} \cup \{\infty\} \longrightarrow \mathbb{CP}^1 \times \mathbb{CP}^1$

$$(x_0 : x_1) \longmapsto \big((x_0 : x_1) , (n x_1 : x_0) \big).$$

In an affine chart $x = \frac{x_1}{x_0}$, $\quad x \mapsto \left(x, \frac{1}{nx}\right)$ (extended to $0, \infty$).
$\mathbb{C} \longrightarrow \mathbb{C} \times \mathbb{C}$



Away from 0, uniform convergence to $x \mapsto (x, 0)$ so the limit seems to be one line, but if we reparametrize $\tilde{x} = nx$, we get $\tilde{x} \mapsto \left(\tilde{x}/n , \frac{1}{\tilde{x}} \right)$. This converges uniformly away from $\tilde{x} = \infty$ to $\tilde{x} \mapsto \left(0, \frac{1}{\tilde{x}}\right)$, so we get the second coordinate axis.

So the limit of u_n is a map  $\longrightarrow \mathbb{CP}^1 \times \mathbb{CP}^1$.

<u>Next time</u>: we'll see that there can be disc bubbles too.

06/04/16 <u>Last time</u>: statement of Gromov's compactness theorem.

We will restate it, this time for Σ with boundary.

<u>Theorem</u>: Suppose $\Sigma_n = (\Sigma, j_n)$ has boundary components $\partial_1\Sigma, \dots, \partial_k\Sigma$ and is equipped with "marked points" on Σ (possibly on $\partial\Sigma$). Suppose $u_n : \Sigma_n \to (X, J_n, \omega)$ is a sequence of J-hol. maps satisfying some Lagrangian boundary conditions $u_n(\partial_i \Sigma)^{(*)} \subseteq L_i$ Lagrangians in X, and with energy $E(u_n) < K$ independent of n.
Then $\exists\, \Sigma_\infty$ "nodal surface" and a subsequence of u_n's converging to a <u>stable</u> J-holomorphic map $u_\infty : \Sigma_\infty \longrightarrow X$ (with Lagrangian boundary conditions, etc, as before).

(*) rather, each component of $\partial_i \Sigma \setminus$ boundary marked points is sent to a Lagrangian in X.

<u>ex</u>:  $\text{im}(\Sigma_n)$ could converge to