08/04/16: <u>Today</u>: * remarks on stable maps — (a) invariance

* Floer's equation vs J-hol strips: — (b) Floer's thm: $HF^*(L,L) \overset{\text{sometimes}}{\cong} H^*(L)$

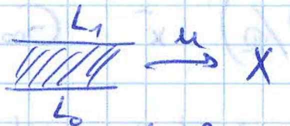
(* Gradings) — (c) Oh spectral sequence

A J-hol map $u : \Sigma_\infty \to X$ (Σ_∞ nodal, with marked points) is called a prestable map; it is stable if its automorphism group is finite, where a automorphism is

$$\begin{tikzcd} \Sigma_\infty \arrow[rr, "f"] \arrow[dr, "u"'] & & \Sigma_\infty \arrow[dl, "u"] \\ & X & \end{tikzcd}$$

where f is the underlying automorphism of trees + biholomorphic on components, preserve marked points.

Rem: if $u: \Sigma_\infty \to X$ is constant, then stability of u implies stability of the domain Σ_∞ (equipped with marked points). If u is constant, then u is stable $\iff$ Σ_∞ is stable.

ex: 

non-constant is stable, because any automorphism $\phi : \mathbb{R}\times[0,1] \circlearrowleft$ is translation, and $u\circ\phi \neq u$.

Rem: in Gromov's compactness, bubbles are only well defined up to overall automorphism $\Rightarrow$ get an element of $\mathcal{M}_{\text{stable}}/\text{Aut of domains}$.
"Aut of domains" has a nearly free action when the maps are stable.

<u>Floer's equation</u>:

It is helpful to have a more flexible way of constructing Floer homology groups, using an auxiliary Hamiltonian.

Define, for $H : [0,1]\times X \to \mathbb{R}$ Hamiltonian, J a.c.s, L_0 and L_1: $CF^*(L_0, L_1; H, J)$, defined whenever $\phi_H^1(L_0) \pitchfork L_1$.

Let $\mathcal{X}^H_{L_0,L_1} := \{$time-1 chords of X_H from L_0 to L_1
$\gamma : [0,1] \to X$, $\gamma(i) \in L_i$ for $i \in \{0,1\}$, $\dot\gamma = X_H\}$.

Define $CF^*(L_0, L_1; H, J) = \Lambda^{|\mathcal{X}^H_{L_0,L_1}|}$.

Differential: for $x^+, x^- \in \mathcal{X}^H_{L_0,L_1}$, $\beta \in \pi_2(x^+, x^-, \ldots)$.

$$\mathcal{M}_H(x^+,x^-,\beta) \begin{cases} u : \mathbb{R}\times[0,1] \longrightarrow X & S := \mathbb{R}\times[0,1] \quad (S,j) \\ u(s,i) \in L_i & \\ \lim_{s\to\pm\infty} u(s,t) = x^{\pm}(t) & \\ \partial_s u + J(\partial_t u - X_H) = 0 : \text{ Floer's equation} & \\ E(u) = \int_S |\partial_t u - X|^2 = \int_S (u^*\omega - d(u^*H\,dt)) < \infty . & \end{cases}$$

Coordinate free Floer's equation: $(du - X_H \otimes dt)^{0,1} = 0$ $\leftarrow$ wrt J, j

where for $f \in \mathrm{Hom}(TS, u^*TX)$, have $(f)^{0,1} = \frac{1}{2}(f + J \circ f \circ j)$.

These are manifolds of index $\mathrm{ind}(\beta)$ for generic J; look at $\mathcal{M}_H(x^+,x^-,\beta)/\mathbb{R}$ when $\mathrm{ind}(\beta) \geq 1$. Then Gromov compactify.

Define $d(x^+) = \sum_{\substack{x^-,\beta \\ \text{st } \mathrm{ind}(\beta)=1}} T^{E(\beta)} \#\left(\mathcal{M}_H(x^+,x^-,\beta)/\mathbb{R}\right) \cdot x^-$. Gromov compactness and gluing imply $d^2 = 0$ (in absence of bad bubbles).

Note: given $u \in \mathcal{M}_H(x^+,x^-)$, gauge transform $\tilde{u}(s,t) = \phi_H^{1-t} u(s,t)$

Then, $\tilde{u}$ is a solution to $\bar{\partial}_{\tilde{J}} \tilde{u} = 0$ with boundaries on $\phi_H(L_0), L_1$, asymptotic to $x^+(1), x^-(1)$, where $\tilde{J} = (\phi_H^{1-t})_* J (\phi_H^{1-t})_*^{-1}$ depends on t (in general, we may have needed t-dependent J anyway for transversality).

$\Rightarrow$ ~~k~~ $CF^*(L_0, L_1; H, J) \cong CF^*(\phi_H(L_0), L_1; H, \tilde{J})$ as chain complexes.

As an example of application, we look at continuation maps $CF^*(L_0, L_1; H_0, J) \to CF^*(L_0, L_1, H_1, J)$. If these are quasi-isos, then $HF^*(\phi_H(L_0), L_1) \cong HF^*(L_0, L_1)$.

Define $\bar{\partial}_{J,H} u := (du - X_H \otimes dt)^{0,1}$ $\leftarrow$ for J.