$\bar{\partial}$ equation $\partial_s u + J \partial_t u = 0$.

Given $x_1 \in CF^*(L_0, L_1, J_0)$, $x_2 \in CF^*(L_1, L_2; J_1)$ and $x_{out} \in CF^*(L_0, L_2; J_2)$, get $\mathcal{M}(x_{out}; x_1, x_2)$ (make some choices of ends, J restricting to J_0, J_1, J_2 on various ends, etc). A count of index 0 solutions will give matrix coefficients for the map $[\mu^2]$ on chain level.

13/04/16 Before continuing talking about product structures, let's talk about signs.

Orientations: [Floer-Hofer (Ham. Floer homology), de Silva + Fooo (Lagr. setting), Seidel, Wehrheim-Woodward "Orientations...", Abouzaid's monograph]

Want: to associate a signed count $\#(\mathcal{M}(p,q)/\mathbb{R})$ in $\partial: CF^*(L_0, L_1)^{\circlearrowleft}$.

More generally, we want an orientation of $\mathcal{M}(p,q)/\mathbb{R}$ or by trivializing the $\mathbb{R}$-action, orient $\mathcal{M}(p,q)$.

Meta-statement (not the most general)$^{\circledast}$: $\mathcal{M}(p,q)$ is canonically oriented, relative to certain orientations of 1-dim vector spaces associated to p, q (the orientation lines o_p, o_q), after fixing spin structures on L_0 and L_1.

$\circledast$ $L_0, L_1 \subseteq X$, $p, q \in L_0 \pitchfork L_1$, say L_0, L_1 oriented.

Rem: $spin(n) \xrightarrow{2:1} SO(n)$, is the universal cover if $n \geq 3$.

Rem: more generally, could ask L_i to be relatively Pin, rel $b \in H^2(X; \mathbb{Z}_2)$.

Spin corresponds to $b = 0$ and L_i oriented as well, condition $b|_L = w_2(L)$.

So, Floer cohomology (rel b) with signs & grading has objects $(L, P$ spin structure rel b, $\tilde{\alpha}_L$ grading, ...$)$.

Notation: V finite dimensional vector space $\rightsquigarrow \det(V) = \underbrace{\Lambda^{top} V}_{\lambda(V)}$, $\mathbb{Z}/2$-graded 1-dimensional vector space, grading is $\dim(V)$.

For M a manifold, get $\lambda(TM)$ the determinant line bundle.
An orientation of $M \leftrightarrow$ trivialization of $\lambda(TM)$.

For $D: X \to Y$ a Fredholm operator, get $\det(D) = \lambda(\operatorname{coker} D)^* \otimes \lambda(\ker D)$,
so get a natural line bundle $\underline{\det} \to \mathrm{Fred}(X, Y)$.

More specific to our setting: $\begin{array}{c}\mathcal{E}\\ \pi\downarrow\uparrow s\\ \mathcal{B}\end{array}$ Banach bundle, with s a section whose linearization is Fredholm.

So, get $\begin{array}{c}\underline{\det}\\ \downarrow\\ \mathcal{B}\end{array}$ and $\begin{array}{c}\underline{\det}\\ \downarrow\\ \mathcal{M} = s^{-1}(0)\end{array}$, $\underline{\det}_u \cong \det(D s_u^{vert})$

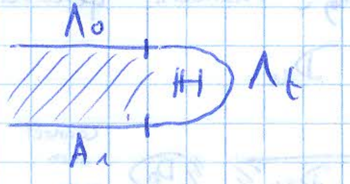
<u>Note</u>: if $\mathcal{M} = s^{-1}(0)$ is transversally cut out, then $\underline{\det}_u \cong \lambda(\ker D s_u^{vert}) = \lambda(T_u \mathcal{M})$

So, an orientation/trivialization of $\begin{array}{c}\underline{\det}\\ \downarrow\\ \mathcal{M}\end{array}$ is the same as an orientation of $\mathcal{M}$.

The orientation line of $p \in L_0 \cap L_1$ (or more generally, $L_0 \xrightarrow{\gamma} L_1$ time 1 chord of X_H): the idea is, given p, construct a (local linearized) Cauchy-Riemann operator D_p associated to p, and take $o_p := \det(D_p)$.

$p \rightsquigarrow$ a bundle pair (E, F). Fix a
$$(\mathbb{D}^2 \setminus \{+1\}, \partial(\mathbb{D}^2 \setminus \{+1\})) \cong (\mathbb{H}, \mathbb{R})$$
negative strip-like end around $+1$ $(\cong +\infty)$: 

E is the trivial bundle $\mathring{\mathbb{D}}^2 \times T_p X \cong \mathring{\mathbb{D}}^2 \times \mathbb{C}^n$, and F is a path of Lagrangian subspaces between $\Lambda_0 := T_p L_0$ and $\Lambda_1 := T_p L_1$, constant near ∞:



Rem: if the L_i's come equipped with a grading structure, e.g. lifts $\begin{array}{ccc} & \tilde{\alpha}_L \nearrow & \tilde{\mathcal{L}}_L \\ & & \downarrow \\ L_i & \longrightarrow & \mathcal{L}_L \end{array}$, then Λ_t is uniquely determined up to homotopy rel endpoints.