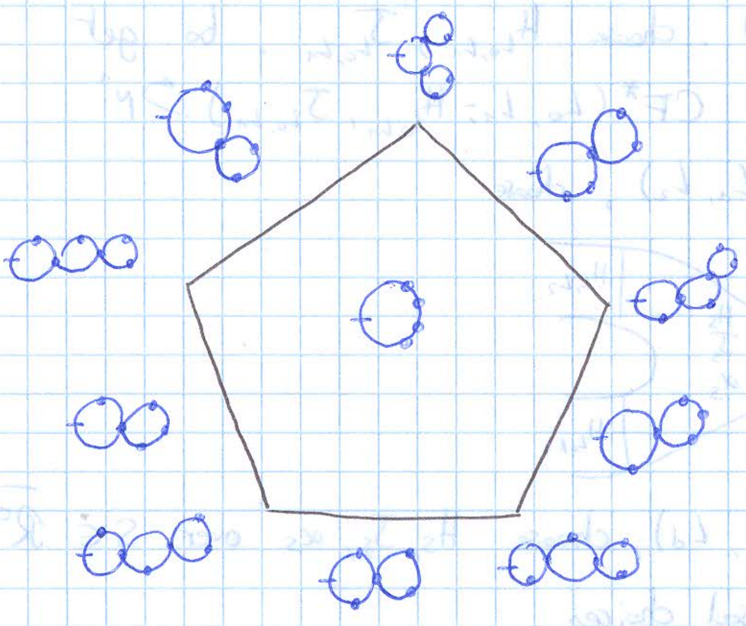
27/04/16: Here is a picture of the Stasheff polytope $\overline{\mathcal{R}}^4$:



Let Λ be a field.

Definition: a ($\mathbb{Z}$-graded) A_∞-category (over Λ) $\mathcal{C}$ consists of the following data:

* a set of objects $\operatorname{ob}\mathcal{C}$
* for any $K, L \in \operatorname{ob}\mathcal{C}$, a $\mathbb{Z}$-graded vector space $\hom^\bullet_{\mathcal{C}}(K, L)$
* for $k \geq 1$ and any $(k+1)$-tuple $(L_0, \dots, L_k)$ in $\operatorname{ob}\mathcal{C}$, "higher composition map":

$$\mu^k : \hom_{\mathcal{C}}(L_{k-1}, L_k) \otimes \dots \otimes \hom_{\mathcal{C}}(L_0, L_1) \to \hom_{\mathcal{C}}(L_0, L_k)$$

of degree $2-k$, satisfying A_∞-relations: for each $d \geq 1$,

$$\sum_{i,s} (-1)^{*_i} \mu^{d-s+1}(x_d, \dots, x_{s+i+1}, \mu^s(x_{s+i}, \dots, x_{i+1}), x_i, \dots, x_1) = 0.$$

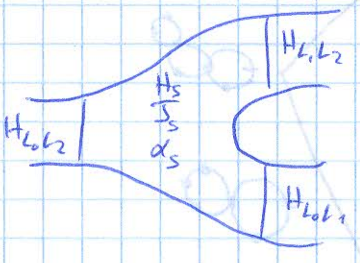
We showed:

Theorem: for (X^{2n}, ω) symplectic satisfying $(*)$ (and maybe $2c_1(X) = 0$ and fixed $\tilde{\mathcal{L}} \to \mathcal{L} \to X$), we can make choices to define an A_∞ cat. over Λ whose objects are Lagrangian branes $L = (L, \tilde{\alpha}_L, P)$ satisfying $(**)$. ~~be~~ ~~get~~ called $\operatorname{Fuk}_S(X)$

S = set of choices

What choices?

* for each (L_0, L_1), choose H_{L_0,L_1}, J_{L_0,L_1}, to get
  $\hom_{\mathcal{F}_S}(L_0, L_1) := CF^*(L_0, L_1; H_{L_0,L_1}, J_{L_0,L_1}) \circlearrowleft \mu^1$
* for each (L_0, L_1, L_2), choose



* for each $(L_0, \dots, L_d)$, choose H_S, J_S, α_S over $S \in \overline{\mathcal{R}}^d$,
  consistent universal choices.

Rem. A_∞ categories do not induce ordinary categories (ie $\exists$ no forgetful functor), before taking H^0.

Definition: an A_∞-functor $F : \mathcal{E} \to \mathcal{D}$ between A_∞-categories is the following data:

* a map $F : \mathrm{ob}\,\mathcal{E} \to \mathrm{ob}\,\mathcal{D}$
* for all $d \geq 1$ and $(L_0, \dots, L_d)$ in $\mathrm{ob}\,\mathcal{E}$, a linear map
  $F^d : \hom_{\mathcal{E}}(L_{d-1}, L_d) \otimes \dots \otimes \hom_{\mathcal{E}}(L_0, L_1) \to \hom_{\mathcal{D}}(FL_0, FL_d)$
  of degree $1-d$ satisfying the A_∞-functor relations for each d and $x_d, \dots, x_1$ composable morphisms in $\mathcal{E}$,

$$\sum (-1)^{*_i} F^{d-s+1}(x_d, \dots, x_{i+s+1}, \mu^s_{\mathcal{E}}(x_{i+s}, \dots, x_{i+1}), x_i, \dots, x_1)$$
$$= \sum_{\ell, i_1, \dots, i_\ell} \mu^\ell_{\mathcal{D}}(F^{i_1}(x_d, \dots), F^{i_2}(\dots), \dots, F^{i_\ell}(\dots, x_1))$$

Let us examine that

* $d=1$: $F^1 \circ \mu^1_{\mathcal{E}} = \mu^1_{\mathcal{D}} \circ F^1$, i.e. F^1 is a chain map.
* $d=2$: $F^1 \mu^2_{\mathcal{E}}(x_2, x_1) - \mu^2_{\mathcal{D}}(F^1(x_2), F^1(x_1)) = \pm F^2(\mu^1_{\mathcal{E}}(x_2), x_1) + F^2(x_2, \mu^1_{\mathcal{E}}(x_1))$
  $+ \mu^1_{\mathcal{D}} F^2(x_2, x_1)$,

i.e. $[F^1]$ intertwines compositions: $[F^1][\mu^2_{\mathcal{E}}](\alpha_2, \alpha_1) = [\mu^2_{\mathcal{D}}]([F^1]\alpha_2, [F^1]\alpha_1)$.