These μ° are supposed to "count disk bubbles".

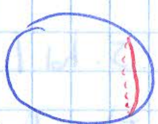
Definition: (X^{an}, ω) is (positively) monotone if $[\omega] = \tau c_1(X)$, $\tau > 0$.
$L \subseteq (X^{an}, \omega)$ is monotone if $[\omega] = \lambda [\mu]$ $\lambda > 0$, where we think of $[\omega], [\mu] : \pi_2(X, L) \longrightarrow \mathbb{R}$.

ex: • $(\mathbb{CP}^n, \omega_{FS})$

• Fano variety: X complex projective variety with $-K_X$ ample, with $i^*\omega_{FS}$ coming from $X \overset{i}{\hookrightarrow} \mathbb{P}^N$ with $i^*\mathcal{O}(1) = (K_X)^{-\otimes 5}$.

ex:

monotone



not monotone: 2 disks of $\neq$ areas, although same Maslov index.

04/05/16

Monotonicity.

(X^{an}, ω) is positively monotone if $[\omega] = \tau . c_1(X)$ on S^2 classes for $\tau > 0$.

A Lagrangian is monotone if $[\omega] = \lambda [\mu]$ $(\lambda > 0)$ as homomorphisms $\pi_2(X, L) \to \mathbb{R}$.
symplectic area, Maslov index

Note: L monotone $\Rightarrow$ $[\omega](\underset{\pi_2(X,L)}{u} \# \underset{\pi_2(X)}{v}) = [\omega](u) + [\omega](v)$

$[\mu](u \# v) = [\mu](u) + 2c_1(X).[v]$

$\Rightarrow [\omega](v) = 2\lambda c_1(X)[v]$ $\forall v \in \pi_2(X)$; ~~nearly~~ forces X monotone with $\tau = 2\lambda$.

For L monotone, N_L = minimal Maslov # of L
$= \{\min \mu(\alpha) \mid \alpha \in \pi_2(X, L), \mu(\alpha) > 0\}$

If L is orientable, $N_L \geq 2$, because any path of Lagr. lifts to the (oriented) double cover. From now on, assume $N_L \geq 2$.

Consequences of monotonicity:

1) Holomorphic spheres: if $u : S^2 \to X$ J-hol sphere, then the energy identity $\Rightarrow \int u^*\omega > 0 \Rightarrow c_1(X)[u] > 0$ by monotonicity.

Since any J-hol sphere is a branched cover of a simple (somewhere injective) J-hol sphere ([McDuff - Salamon]), monotonicity implies that Chern 1 spheres are simple, hence transversely cut out (by generic J)

Claim: Chern 1 spheres sweep out a "high codim subset of X".

Namely, when $c_1(A) = 1$,

$$\mathcal{M}_1(A) := \{ u : (S^2, x_0) \to X \text{ J-hol} \mid u_*[S^2] = A \} / \text{reparam}$$

is a manifold (for generic J) of dimension

$$2n + 2\underbrace{c_1(A)}_{=1} + \underset{\substack{\uparrow \\ \text{marked pt}}}{2} - \underset{\substack{\uparrow \\ \text{reparam}}}{6} = 2n - 2$$

We have $ev : \mathcal{M}_1(A) \to X : u \mapsto u(x_0)$. The point is that codim ≥ 2.

We'll use this to argue that on $\partial($ index 2$)$, sphere bubbling does not occur.

[**Lemma**: on $\partial($ ind 2 $)$, sphere bubbling does not occur.

Proof: the only possible bubbling is 

, by count of dimension. □

2) We can count the holomorphic discs: [Lazzarini, Kwon-Oh]

(// McDuff's result) $\Rightarrow$ Maslov 2 hol discs with boundary on monotone L are simple, hence transversely cut out.

Given a class $\beta \in \pi_2(X, L)$ with $\mu(\beta) = 2$, let

$$\mathcal{R}^0(L; \beta, J) = \{ u : (\mathbb{D}^2, \partial\mathbb{D}^2, \underset{\partial\mathbb{D}^2}{p}) \to (X, L, L) \text{ J-hol} \} / \text{Aut. of domain}.$$

By Lazzarini, Kwon-Oh, for generic J, this is a manifold of dim

$$n + \underbrace{\mu(\beta)}_{=2} + \underset{\substack{\uparrow \\ \text{marked pt}}}{1} - \underset{\substack{\uparrow \\ \text{reparam}}}{3} = n$$