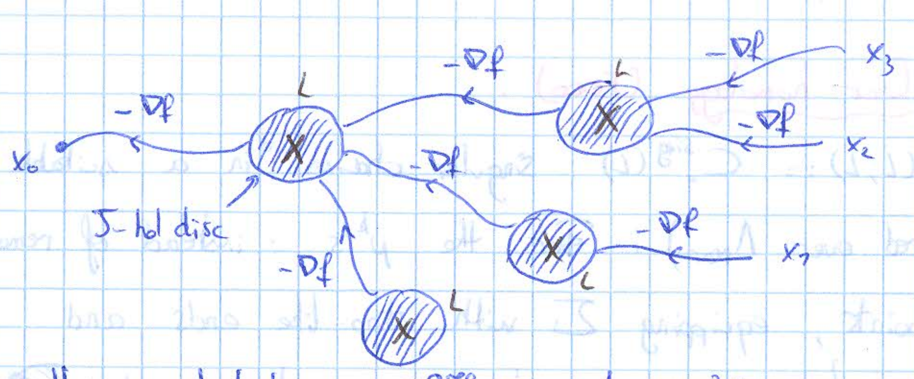


this contributes $T^{\text{area}} x_0$ to $\mu^3(x_3, x_2, x_1)$.

Bubbling of discs is no longer a boundary in these moduli spaces $\rightsquigarrow$ broken trajectories are boundaries.

09/05/16

<u>One example</u> (very sketchy) of a calculation of monotone $HF^*(L,L)$ with the A_∞-structure: $T^n_{Cl} \subset \mathbb{P}^n$ the Clifford torus.

Define $T^{n+1} = \left(S^1\left(\frac{1}{\sqrt{n+1}}\right)\right)^{n+1} \subseteq S^{2n+1}(1) \subseteq \mathbb{C}^{n+1}$. Then, $T^n_{Cl} := T^{n+1}/S^1_{\text{diagonal}} \subseteq S^{2n+1}/S^1 \cong \mathbb{CP}^n$.

[Oh]: $L := T^n_{Cl}$ is monotone, wrt ω_{FS}, and $N_L = 2$. This follows from the analysis of ω and μ on $\pi_2(X, L)$, via

$$0 \to \pi_2(X) \to \pi_2(X, L) \to \pi_1(L) \to 0.$$

ex: presenting $\mathbb{CP}^2 = \mathbb{C}^2 \cup H_\infty$, T^2 is a standard product torus in each $\mathbb{C}$-coordinate, of radius $\frac{1}{\sqrt{3}}$.

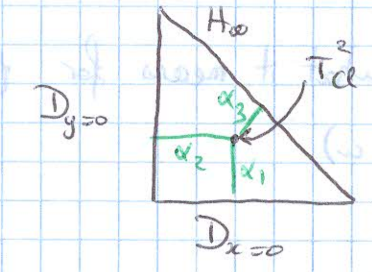
There is a canonical choice of Spin structure on T^n_{Cl}, coming from the canonical trivialization $TT^n \cong T^n \times \mathbb{R}^n$.

Or, [Cho] analyzed effect of changing Spin structure on your result.

<u>Theorem</u> [Cho] Using the standard integrable J, there are $n+1$ families of Maslov 2 discs.

ex: 

$\hat{T}_{cl} \subset \mathbb{P}^1$;



In $\mathbb{P}^2$, using a "toric model": the polytope is a base of a torus fibration $\mathbb{P}^2 \to \Delta$.

We can compute explicitely the homology classes in $H_1(T^n)$ the boundaries of these discs live in: basically the images of $\tilde{\alpha}_1, \dots, \tilde{\alpha}_{n+1} \in H_1(T^{n+1})$ (generators), projected to $H_1(T^n)$.

$\Rightarrow$ Given a rank 1 local system ∇ on T^n with holonomies (in $\mathbb{C}^*$) equal to $x_1, \dots, x_n, \dfrac{1}{x_1 \cdots x_n}$ on $\mathrm{im}(\tilde{\alpha}_1), \dots, \mathrm{im}(\tilde{\alpha}_{n+1})$.

$$\Rightarrow m_0(L, \nabla) := x_1 + \dots + x_n + \frac{1}{x_1 \cdots x_n}.$$

<u>Rem</u>: can think of $m_0(L,\nabla)$ as being part of a function (as ∇ varies)

$W: (\mathbb{C}^*)^n \to \mathbb{C} : \nabla \mapsto m_0(L,\nabla)$ "Maslov 2 disc superpotential"

$(\mathbb{C}^*)^n$: space of rank 1 local systems on L.

<u>Theorem</u>: [Cho] There are exactly $(n+1)$ rank 1 local systems on L with $HF^*((L,\nabla),(L,\nabla)) \neq 0$. (They are $\simeq H^*(T^n)$; the others are 0)

In fact, there is a bijection between $\{\nabla \mid HF^*(L,\nabla) \neq 0\}$ and $\mathrm{crit}(W)$.

$\leadsto$ $\partial_i W$ (partial derivatives) have something to say about μ^1.

And, we can compute μ^2, etc, via higher partial derivatives of W.

This is a special partly worked out case of HMS:

$$\mathcal{F}uk(\mathbb{P}^n) \longleftrightarrow MF\left((\mathbb{C}^*)^n, W = z_1 + \dots + z_n + \frac{1}{z_1 \cdots z_n}\right)$$

MF: "matrix factorizations"

$(T^n_{cl}, \nabla_i) \longleftrightarrow \mathcal{O}_{p_i}$ structure sheaf at a critical point

It will turn out that $\coprod_{i=1}^{n+1} (T^n_{cl}, \nabla_i)$ (split-)generate the monotone Fukaya category of $\mathbb{P}^n$.