Draw γ: for every $L \subseteq Z$, get a generalized thimble Δ^L.
Note that it seems that

$$\mathrm{Hom}(\Delta^L, \Delta^K) := HF^\bullet(\phi_\varepsilon \Delta^L, \Delta^K) \cong \underset{\text{in } Z.}{HF^\bullet}(L, K)$$

Theorem [Abouzaid-Auroux-Katzarkov] This is true (with signs) (for Lagrangian branes when $\nu(Z)$ (normal bundle) is spin.

Proposition. [AAK, A-Ganatra] there is a fully faithful A_∞-embedding

$$\mathcal{F}(Z) \xhookrightarrow[\Delta^{(\cdot)}]{} \mathcal{F}(E, W)$$

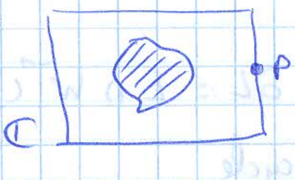
Theorem: $\Delta^{(\cdot)}$ (any split-generating collection in Z in sense of satisfying "Abouzaid's generation criterion") split-generates.

↳ strictly stronger than split-generation.

__Next time:__

$$\phi_{2\pi} \circlearrowright \mathcal{F}(E, W) \underset{\cup}{\overset{\cap}{\rightleftarrows}} \underset{\text{general fiber}}{\mathcal{F}(M)} \circlearrowleft \mu \text{ monodromy map}$$

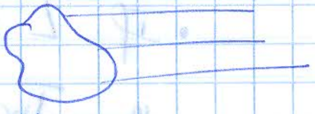
23/05/16: __Last time:__ (E, W) symplectic LG-model, e.g. a symplectic fibration with singularities. Take p near ∞, then the "general fiber" $M := W^{-1}(p)$ is a symplectic submanifold.



↳ Get $\mathcal{F}(E, W)$ and $\mathcal{F}(M)$.

__Today:__ How are these two related?

__Recall:__ ob $\mathcal{F}(E, W)$ = W-admissible Lagrangians,



with $D_L \subseteq \mathbb{R}$ the set of heights. On cohomology,

$$\mathrm{Hom}_{\mathcal{F}(E,W)}(K, L) = HF^\bullet(\phi_{\varepsilon_K} K, L)$$

↑ sufficiently large "admissible positive flow"

The cohomology of the other definition (with oriented stuff) always coincides with this.

<u>Idea</u>: can define functors, which are adjoint:

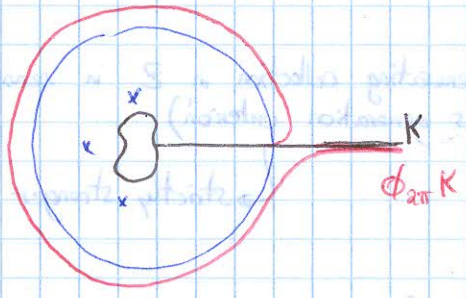
$$\phi_{2\pi} \circlearrowright \mathcal{F}(E,w) \underset{\cup_{ccw}}{\overset{\cap}{\rightleftarrows}} \mathcal{F}(w) \circlearrowleft \mu$$

and $\cup_{cw}$ is drawn as a second arrow $\mathcal{F}(w) \to \mathcal{F}(E,w)$.

$\cap$ is "cap", same intersection.

$\cup_{ccw}$ and $\cup_{cw}$ are the Orlov functors, counterclockwise & clockwise

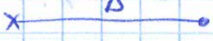
* μ is the global monodromy, induced by // transport around large loop.
* $\phi_{2\pi}$ is "once wrapping, counterclockwise":



In general, there is a $\phi_{2\pi s}$ $\forall s \in \mathbb{Z}$.

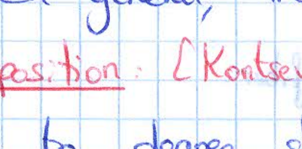
[<u>Proposition</u>: [Kontsevich, Seidel] $\phi_{-2\pi}$ is the "Serre functor", up to degree shift.

* $\cap : \mathcal{F}(E,w) \to \mathcal{F}(M)$ is "intersection with a fiber"
  [Abouzaid-Seidel, Abouzaid Ganatra].
  - If L has 1 horizontal end, $\cap : L \longmapsto \partial L := L \cap w^{-1}(p)$



$\longmapsto V = \partial\Delta$ vanishing cycle

  - If L has multiple ends, we should think of $\cap$ as landing in $Tw\,\mathcal{F}(M)$, $Perf\,\mathcal{F}(M)$, ...



$\longmapsto \mathrm{Cone}\left(\partial^1 L \xrightarrow{\alpha} \partial^2 L\right)$,

where α is some morphism coming from count of discs in the total space

• No end: L maps to $0 \in Perf$.

[cf Biran-Cornea's work on lagr. cobordisms]