Returning to LG models (E, w), there is a map

$$HH_*(\mathcal{F}(E,w), \mathcal{B}_{\phi_{2\pi}}) \xrightarrow[\mathcal{OE}_w]{} HF^*(E,w)$$

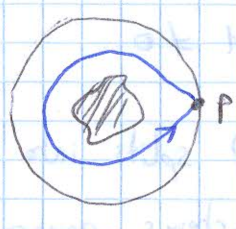
Theorem [Abouzaid-Ganatra] if $\mathcal{OE}_{w|\mathcal{A}}$ hits 1, $\mathcal{A}$ split-generates.

Expectation: $\mathcal{OE}_w$ is always an isomorphism, at least when W is a Lefschetz fibration (true with 1 critical point, ...)

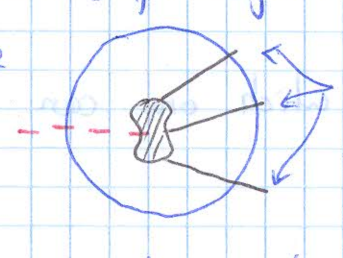
03/06/16

Generating Fukaya categories of LG models

Setup: (E^{2n+2}, W) symplectic LG model: $W: E \to \mathbb{C}$, with $M := W^{-1}(p)$ general fiber and $\mu: M \to M$ symplectic monodromy induced by



$\hookrightarrow \mathcal{F}(E,W)$ has for objects Lagrangian branes, such that $W(L)$ looks like



Last time: generation criterion: X^{2n+2} compact symplectic, we can construct $\mathcal{OC}: HH_{*-n}(\mathcal{F}(X)) \to QH^*(X)$

Theorem: [Abouzaid-Ganatra] If $\mathcal{OC}|_{\mathcal{A}}$ hits $1 \in QH^*(X)$, then $\mathcal{A}$ split-generates $\mathcal{F}(X)$.

Theorem: under the above hypotheses, $\mathcal{OC}$ and

$$\mathcal{CO}: QH^*(X) \longrightarrow HH^*(\mathcal{F}(X))$$

are isomorphisms, preserving various structures.

Want: a similar criterion to this one, for $\mathcal{F}(E,w)$

Application: (some still in progress, or conjectural)

* Show a basis of thimbles (or some other collection of Lagrangians) split-generates.
* Give a geometric description of the Hochschild invariants of $\mathcal{F}(E,w)$ $\rightsquigarrow$ new invariants of (E,w), which can be used to detect various phenomena (symplectic invariants of singularities of polynomials).

Problems. (1) $HH_*(\mathcal{F}(E,w))$ is the wrong place for a generation criterion. (homology)

ex: for $\mathcal{A} \subseteq \mathcal{F}(E,w)$ a basis $\{\Delta_1, \dots, \Delta_k\}$ of thimbles, we showed that a minimal model of $\mathcal{A}$ looks like:

$$\hom_{\mathcal{A}}(\Delta_i, \Delta_j) = \begin{cases} HF^*(V_i, V_j) & i<j \\ \mathbb{k} & i=j \\ 0 & i>j \end{cases}$$

⚠ need cyclic chains in CC

(*) $\Rightarrow HH_*(\mathcal{A}) = \bigoplus_{i=1}^{n} \mathbb{k}$ ↪ corresponds to $\hom_{\mathcal{A}}(\Delta_i, \Delta_i)$

We get this because for $\mathcal{A}$ strictly unital augmented, there is a reduced version of CC

$$CC^{red}(\mathcal{A}) = \bigoplus_{X_0, \dots, X_k} \hom_{\mathcal{A}}(X_k, X_0) \otimes \overline{\hom_{\mathcal{A}}(X_{k-1}, X_k) \otimes \dots \otimes \hom_{\mathcal{A}}(X_0, X_1)}$$

↪ in reduced thing

The only place we can have a unit is the 1^{st} one, hence (*).

(2) "Closed string invariant" of (E,w) (analogue of $QH^*(X)$). Ideally, it should be a unital ring, meaning it has a "1", and maybe it should be $\cong HH^*(\mathcal{F}(E,w))$ the cohomology (we just saw it can't be the homology), which is a unital ring too.

Conjecture of Seidel (2001): let w Lefschetz, $\mathcal{A}$ any basis of thimbles; there should be a LES