Math 51 Homework 3

Due Friday July 8, 2016 by 1 pm

Instructions: Complete the following problems. Late homework will not be accepted. Please be sure to review the expectations for your submitted homework outlined online (such as: always including your name and ID number on the homework, stapling your homework, and guidelines for write-ups which will receive full credit). *Make sure to submit your homework to the correct person; (if you are in Section 01, submit to Zev, and if in Section 02, submit to Valentin).*

Remark: The following problems may be helpful in preparing for the Midterm exam: the book problems in Section 4, 11, 12, 13.

Part I: Book problems: From Levandosky's *Linear Algebra*, do the following exercises:

- Section 4: # 10
- Section 11: # 4, 8, 15
- Section 12: # 3, 7, 13acd* (*Note: for each designated part of # 13, either explain briefly why the statement is "true", or give a counterexample if the statement is "false.")
- Section 13: # 2, 6, 8, 18, 22
- Section 14: # 2, 10, 12
- Section 15: # 1degh, 5, 6

Part II: Non-book problems:

1. *Systems of coordinates*. Suppose V is a subspace of $\mathbb{R}^n$ and $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a basis for V. Then by Propsition 11.1, given any vector $\mathbf{v} \in V$, there are *unique* scalars $c_1, \ldots, c_k$ satisfying

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k.$$

These scalars $c_1, \ldots, c_k$ are called the *coordinates of* $\mathbf{v}$ *with respect to* $\mathcal{B}$ (see also Levandosky portion of your text, p. 145-147).

Turning to some specific examples, by Levandosky Examples 11.1 and 11.2, both

$$\mathcal{S} = \left\{ \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ and } \mathcal{B} = \left\{ \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \right\}$$

are bases for $\mathbb{R}^3$.

(a) Find the coordinates of $\mathbf{v} = \begin{bmatrix} 8 \\ 5 \\ -2 \end{bmatrix}$ with respect to $\mathcal{S}$.

(b) Find the coordinates of $\mathbf{v} = \begin{bmatrix} 8 \\ 5 \\ -2 \end{bmatrix}$ with respect to $\mathcal{B}$.

(c) Find the coordinates of $\mathbf{w} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ with respect to $\mathcal{B}$. (*Hint*: can you avoid computation here?)

2. Suppose $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^n$ which are *mutually orthogonal*. By L4 Exercise 4.10 (which you solved earlier in this HW), the set $\mathcal{B} = \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. In the language of bases, that means $\mathcal{B}$ is a basis for the subspace

$$V = \operatorname{span}(\mathbf{u}, \mathbf{v}, \mathbf{w}) \subset \mathbb{R}^n.$$

Use dot products to find a formula for the coordinates of a vector $\mathbf{x} \in V$ with respect to $\mathcal{B}$; that is, find expressions for the unique scalars c_1, c_2, c_3 satisfying

$$\mathbf{x} = c_1\mathbf{u} + c_2\mathbf{v} + c_3\mathbf{w}$$

involving dot products of these vectors.

3. There are 7000 undergraduate students at Stanford. Let M be the 7000×7000 matrix whose ij entry is 1 if student i and student j are Facebook friends and 0 if they are not. (Here we assume that Facebook does not permit a person to be a friend of themselves, so all the diagonal entries of M are zero.) What information does M^2 contain? That is, what is the meaning of the ij entry of M^2?

4. (a) Find two matrices A, B, with A of size 4×5 and B of size 5×4, where the product AB is a 4×4 matrix of *rank* equal to 4. (*Hint*: try to arrange for AB to be as simple as possible — even the identity matrix I_4 — and with many entries of A and B equal to zero).

(b) Find two matrices C, D with C of size 4×3 and D of size 3×4, where the product CD is a 4×4 matrix of rank equal to 3.

(c) Is it possible to find two matrices E, F with E of size 4×3 and F of size 3×4, where the product EF is a 4×4 matrix of rank equal to 4? If so, find an example; if not, explain why not.