Math 51 Homework 6

Due Friday July 29, 2016 by 1 pm

Instructions: Complete the following problems. Late homework will not be accepted. Please be sure to review the expectations for your submitted homework outlined online (such as: always including your name and ID number on the homework, stapling your homework, and guidelines for write-ups which will receive full credit). *Make sure to submit your homework to the correct person; (if you are in Section 01, submit to Zev, and if in Section 02, submit to Valentin).*

Remark: All of the problems assigned may be helpful in preparing for the Midterm exam.

Part I: Book problems: From **L**evandosky's *Linear Algebra* and **C**olley's *Vector Calculus*, do the following exercises:

- Section **C**2.5, # 4, 6, 26,
- Section **C**2.6: # 4, 8, 12, 20, 22 (**note**: in terms of the language we are using in class, the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ which appear in Colley correspond to $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ respectively. For instance: a vector in $\mathbb{R}^2$ which Colley refers to as $5\mathbf{i}-6\mathbf{j}$ means $5\mathbf{e}_1-6\mathbf{e}_2=\left[\begin{smallmatrix}5\\-6\end{smallmatrix}\right]$, and similarly the vector $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ in $\mathbb{R}^3$ means $2\mathbf{e}_1+3\mathbf{e}_2-4\mathbf{e}_3=\left[\begin{smallmatrix}2\\3\\-4\end{smallmatrix}\right]$.)
- Miscellaneous exercises at the end of Chapter **C**2 (page 183 of Colley): #14, 20, 26

Part II: Non-book problems:

1. (a) Find, using any method you wish (e.g., formula (4) in **C**2.3 for the tangent plane to a graph of a function of two variables, or formula (6) in **C**2.6 for the tangent plane to a level set of a function of three variables) a formula for the tangent plane to the graph of $f(x,y)=x^2+4y^2$ at $(x,y)=(0,3)$.

(b) Find a formula for a tangent plane to the surface $\{x^3+2xy+\cos(zx)=3\}$ at the point $(1,1,\pi/2)$.

2. Consider the equation

$$xz^2+y^2z^5=19. \tag{1}$$

The point $(3,4,1)$ is a solution to this equation, but there are many more solutions. If we treat x and y as independent variables, then in a neighborhood of $(3,4,1)$, Equation (1) determines z as a function of x and y. We say that this function $z(x,y)$ is defined *implicitly* by Equation (1). Writing an explicit formula for this function is difficult, but you can nevertheless compute the linearization of the implicitly-defined function $z(x,y)$ at the point $(3,4)$.

(a) Treating x and y as independent variables and treating z as a function of x and y, differentiate both sides of Equation (1) with respect to x. Solve for $\frac{\partial z}{\partial x}$ and evaluate this function at $(3,4)$. This technique is called *implicit differentiation.*

(b) Use implicit differentiation to compute $\frac{\partial z}{\partial y}(3,4)$.

(c) Write down the linearization $L(x,y)$ of $z(x,y)$ at $(3,4)$.

(d) Use L to approximate the value $z(3.01, 4.02)$. Plug your approximate result into Equation (1) (you may use a computer) and see if it approximately satisfies the equation.