Math 51 Midterm 2 — July 27, 2016

Name: ______________________________ **SUID#:** ______________

Circle your section:	
Section 01 (1:30-2:50PM)	Section 02 (3:00-4:20PM)

- Complete the following problems. In order to receive full credit, please show all of your work and justify your answers. You do not need to simplify your answers unless specifically instructed to do so. You may use any result proved in class or the text, but be sure to clearly state the result before using it, and to verify that all hypotheses are satisfied.
- Please check that your copy of this exam contains 8 numbered pages and is correctly stapled.
- This is a closed-book, closed-notes exam. No electronic devices, including cellphones, headphones, or calculation aids, will be permitted for any reason.
- **You have 1 hour and 20 minutes.** Your organizer will signal the times between which you are permitted to be writing, including anything on this cover sheet, and to have the exam booklet open. During these times, the exam and all papers must remain in the testing room. When you are finished, you must hand your exam paper to a member of teaching staff.
- Paper not provided by teaching staff is prohibited. If you need extra room for your answers, use the back side of the question page or other extra space provided at the front of this packet, and clearly indicate that your answer continues there. Do not unstaple or detach pages from this exam.
- Please show your work unless otherwise indicated. *Do not make multiple guesses: if there are multiple answers given to a question that only asks for one answer, your grade will be the minimum of the possible scores you would have received.*
- It is your responsibility to arrange to pick up your graded exam paper from your section leader in a timely manner. You have only until **one week after your graded midterm is made available**, to resubmit your exam for any regrade considerations; consult one of your CAs for the exact details of the submission process.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the Stanford's honor code with respect to this examination."

Signature: ______________________________

The following boxes are strictly for grading purposes. Please do not mark.

Question:	1	2	3	4	5	6	7	**Total**
Points:	12	12	16	16	15	16	12	99
Score:								

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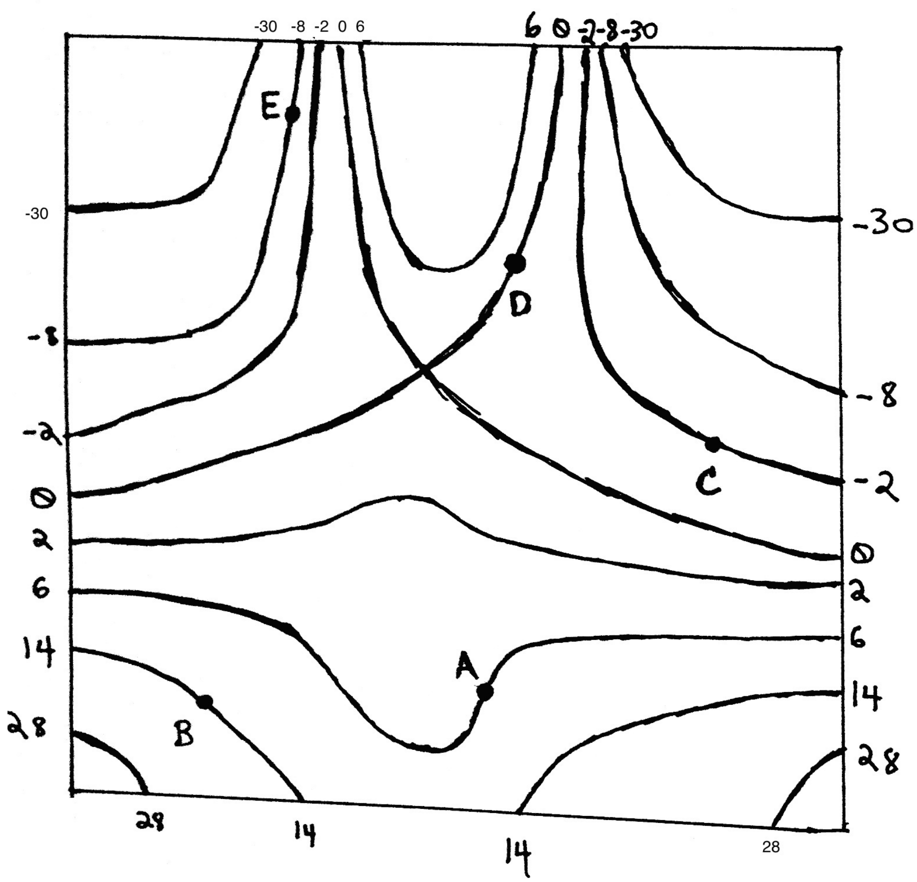
1. (12 points) (2 points each) The diagram below shows several marked points on the contour map of a function $f(x, y)$. You may assume that f has continuous first and second derivatives, and that the scales on the x and y axes, which are parallel to the edges of the box, are the same. The numbers drawn are the heights of the various level sets.



Circle the appropriate word to complete each sentence (there is a unique *best answer* based on the diagram in each case). No justification is necessary.

(a) At the point A the value of $\frac{\partial f}{\partial x}$ is POSITIVE ZERO NEGATIVE.

(b) At the point A, the value of $\frac{\partial f}{\partial y}$ is POSITIVE ZERO NEGATIVE.

(c) At the point B, the value of $\frac{\partial f}{\partial x}$ is POSITIVE ZERO NEGATIVE.

(d) At the point C, the value of $\frac{\partial^2 f}{\partial y^2}$ is POSITIVE NEGATIVE.

(e) At the point D, the value of $\frac{\partial^2 f}{\partial x \partial y}$ is POSITIVE NEGATIVE.

(f) At the point E, the value of $\frac{\partial^2 f}{\partial x^2}$ is POSITIVE NEGATIVE.

2. (a) (6 points) Let $\mathbf{r}(t) = \begin{bmatrix} \frac{1}{1+t^2} \\ \frac{1}{e^{t-1}} \end{bmatrix}$. Find a parametric representation of the tangent line to the image of $\mathbf{r}$ at $\begin{bmatrix} 1/2 \\ 1 \end{bmatrix}$.

(b) (6 points) Find, using a method of your choice, an equation (of the form $ax + by + cz = d$) for the tangent plane to the surface $\{(x, y, z)|z = 2 + x^3 - 2xy^2\}$ at the point $(1, 1, 1)$.

3. Let $Ref_L : \mathbb{R}^2 \to \mathbb{R}^2$ denote the linear transformation which is reflection across the line $L = \text{span}(\left[\begin{smallmatrix} 1 \\ -3 \end{smallmatrix}\right])$.

   (a) (6 points) Let $\mathcal{B} = \{\left[\begin{smallmatrix} 1 \\ -3 \end{smallmatrix}\right], \left[\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}\right]\}$. Find the matrix B for Ref_L in $\mathcal{B}$-coordinates (in class, we called this matrix $[\mathcal{M}(Ref_L)_{\mathcal{B}}]$).

   (b) (6 points) Find, using any method of your choice (but with justification), the matrix A for the linear transformation Ref_L (in standard coordinates).

   (c) (4 points) What is the matrix for the composition $Ref_L \circ Ref_L$?

4. (16 points) Find, with justification, an *orthonormal eigenbasis* for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix}$.

5. Suppose the temperature at a point in space is described by the following function of $\mathbb{R}^3$:

$$T(x, y, z) = x^3y^2 - \cos(x^2z)$$

(a) (6 points) Imagine a bee is flying in space and is at the point $(1, 2, \frac{\pi}{2})$. If the bee is flying in the unit direction $\frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, is the temperature increasing, decreasing, or staying the same?

(b) (4 points) The bee is feeling too hot at $(1, 2, \frac{\pi}{2})$. What (not necessarily unit) direction should the bee fly in to decrease the temperature as quickly as possible?

(c) (5 points) Suppose the *humidity* at a point in space is described by the function

$$H(x, y, z) = x^3 - y^2 + z^2.$$

Set up (but do not solve) for a system of equations (or inequalities) for a unit direction $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ which the bee could fly in from the point $(1, 2, \frac{\pi}{2})$ in which the humidity would stay the same and temperature would increase.

6. Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is the following function:

$$f(x, y) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} e^x \\ e^y + y \end{bmatrix}. \tag{1}$$

You may assume without justification that f is differentiable.

(a) (4 points) Compute the matrix of partial deriatives of f at $(0, 0)$, $Df(0, 0)$.

(b) (6 points) Using the fact that f has a linearization, estimate $f(-0.1, 0.1)$.

(c) (6 points) Let $g : \mathbb{R}^3 \to \mathbb{R}^2$ be a differentiable function, such that $g(1, 1, 3) = (0, 0)$. Suppose that $Dg(1, 1, 3) = \left[\begin{smallmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{smallmatrix}\right]$, calculate $D(f \circ g)(1, 1, 3)$.

7. (12 points) (2 points each) Each of the statements below is either *always true* ("T"), or *always false* ("F"), or *sometimes true and sometimes false, depending on the situation* ("MAYBE"). For each part, decide which and circle the appropriate choice; you *do not* need to justify your answers.

(a) If the partial derivatives of a function $f : \mathbb{R}^n \to \mathbb{R}$ all exist at $\mathbf{a} \in \mathbb{R}^n$, then f is differentiable at $\mathbf{a}$. T F MAYBE

(b) If an $n \times n$ matrix A is diagonalizable, then so is A^3. T F MAYBE

(c) If an $n \times n$ matrix A^2 is diagonalizble, so is A. T F MAYBE

(d) There are functions $f : \mathbb{R}^2 \to \mathbb{R}$ which are not continuous at at $(0, 0)$ but are still differentiable at $(0, 0)$. T F MAYBE

(e) If a 3×3 matrix A only has -1 and 2 as eigenvalues, then A is invertible. T F MAYBE

(f) If a 3×3 matrix A has only -1 and 2 as eigenvalues, then A is diagonalizable. T F MAYBE

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