Math 215B Homework 1

Due January 18th, 2013 by 5 pm

Please remember to write down your name and Stanford ID number (9 digits). All pages and sections refer to pages and sections in Hatcher's *Algebraic Topology*.

1. (6 points) Solve page 19 (§0) problem 2.

2. (6 points) Solve page 38 (§1.1) problem 2.

3. (8 points) Solve page 38 (§1.1) problem 5.

4. (6 points) Solve page 38 (§1.1) problem 6.

5. (6 points) Solve page 38 (§1.1) problem 20.

6. (6 points) Read about the fundamental group of products $\pi_1(X \times Y)$ (page 34, Proposition 1.12), and then solve page 38 (§1.1), problem 10.

7. (6 points) A **retract** of a topological space X onto a subspace $A \subset X$ is a map $f : X \to A$ such that $f|A = id_A$. (Note: A **deformation retract** of X onto A can be thought of as a homotopy from id_X to some retract onto A relative to A, meaning the homotopy does not change the function restricted to A). Read Proposition 1.17 (page 36) which states that if X retracts onto A, then $i_* : \pi_1(A, a) \to \pi_1(X, a)$ is injective, where $i : A \subset X$ is the inclusion and $a \in A$.

   Then, solve page 38 (§1.1), problem 16 parts a and b.

8. (6 points) Solve page 79 (§1.3) problem 3.

9. (10 points) A **topological group** is a group G together with a topology on G such that the operation of multiplication $\times$ and taking inverses are both continuous functions with respect to the topology. In what follows, let G be a topological group with identity element e.
   (a) Given loops $f, g : (S^1, *) \longrightarrow (G, e)$, define a loop $f \star g(t) := f(t) \times g(t)$ using the pointwise product on G. Show that $f \star g \simeq f \cdot g$ via a basepoint-preserving homotopy.
   (b) Show that $\pi_1(G, e)$ is abelian. (Hint: show that $f \star g \simeq g \cdot f$).