Math 215B Homework 4–extended version

Due Monday February 11th, 2013 by 5 pm

Please remember to write down your name and Stanford ID number (9 digits). All pages and sections refer to pages and sections in Hatcher's *Algebraic Topology*.

1. (9 points) *Simplicial homology of the Klein bottle* Solve §2.1 (page 132), problem 5.
2. (9 points) *Simplicial homology of another* Δ*-complex* Solve §2.1 (page 132), problem 6.
3. (8 points) *Retracts.* Solve §2.1 (page 132), problem 11.
4. (8 points) *Exact sequences.* Solve §2.1 (page 132), problem 14.
5. (8 points) *Exact sequences II.* Solve §2.1 (page 132), problem 15.
6. (8 points) *Relative homology, definitions.* Solve §2.1 (page 132), problem 16.
7. (10 points) *Van Kampen's theorem and* H_1.
   a. Let X be path-connected with basepoint, and suppose $X = A \cup B$, such that $x_0 \in A, B$ and $A \cap B$ is path-connected. What does Van Kampen's theorem for the cover imply about $H_1(X)$?
   b. Use this to recompute the singular homology of the Klein bottle in 1, and verify it agrees with the simplicial computation.
8. (6 points) *Computing relative homology.* Solve §2.1 (page 132), problem 17a (just the case $X = S^2$)
9. (7 points) *The homology of suspensions.* The **cone** of a topological space X is defined as

$$CX := X \times [0,1]/X \times \{0\}, \tag{0.1}$$

   where we recall the quotient space Y/A for A a subspace is defined as $Y/A = Y \coprod \{p\}/\sim$, with $p \sim a$ for all $a \in A$. Similarly, the **suspension** SX is defined as

$$SX := X \times [0,1]/(X \times \{0\} \coprod X \times \{1\}). \tag{0.2}$$

   Using what we have learned so far, prove an important relationship between the reduced homology of a space and its suspension, namely that $\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$ for all n. More generally, solve §2.1 (page 132), problem 20.
10. (7 points) *Towards the homology of CW complexes.* Solve §2.1 (page 132), problem 22. This problem begins what will eventually be a more detailed understanding about the homology groups of CW complexes.